The nuclear shell-model: from single-particle motion to collective effects

1. Nuclear forces and very light nuclei

2. Independent-particle shell model and few nucleon correlations

3. Many-nucleon correlations: collective excitations and symmetries

Two major questions to address nuclei within the nuclear shell model

1. How to build up the shell-model basis in an optimal way (computational)

2. How to handle the effective nucleon-nucleon interaction Veff.

Shell-model basis states

- Use explicit construction: coupling (spin,isospin) to state of fixed J, π,T
- Construct Slater determinant states: given M and T_z.

  ex, 4 particles in sd-shell

  | 1d5/2,-1/2; 1d5/2,-3/2; 1d3/2, +3/2; 2s1/2,+1/2>(M=0)

A very usefull approach is a bit-representation, known as the M-scheme.

0	1	1	0	0	0	0	0	0	1	0	1
-5	-3	-1	1	3	5	-3	-1	1	3	-1	1

$=2^1 + 2^3 + 2^{10} + 2^{11} = 3082$

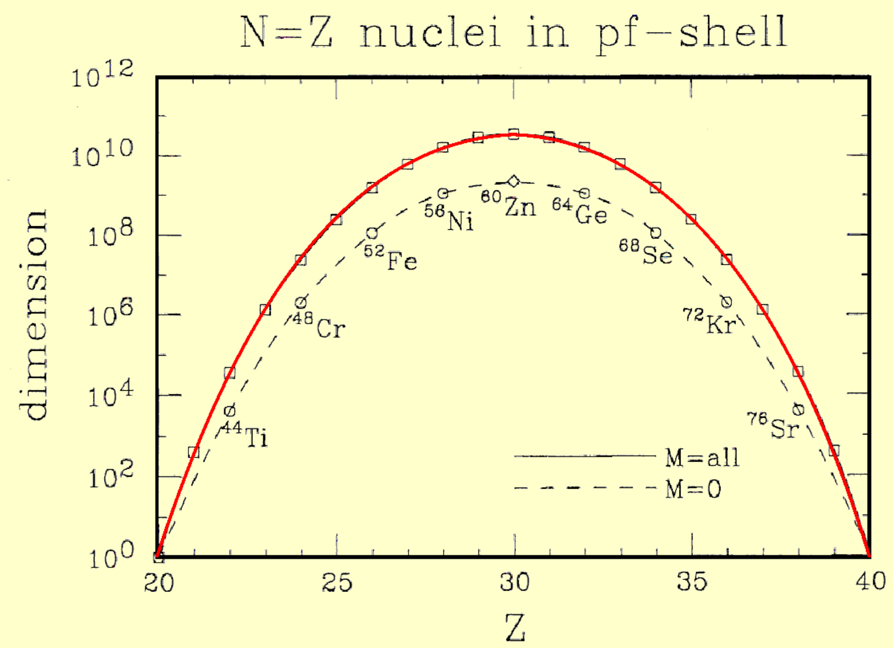
$1d_{5/2}$ (first six), $1d_{3/2}$ (next four), $2s_{1/2}$ (last two)

- Counting # basis states (approx.)

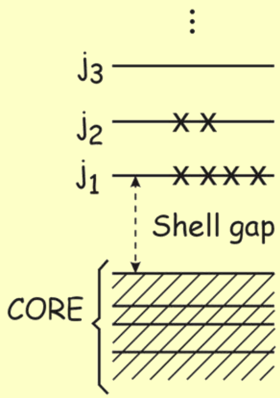
n particles, N orbitals $\simeq \binom{N_p}{n_p}\binom{N_n}{n_n}$

ex. ^{60}Zn(fp):10p, 10n;20 orbitals

$\binom{20}{10}\binom{20}{10} \simeq 3.4.10^{10}$



NUCLEAR RESIDUAL INTERACTIONS...



SEPARATION OF A NUCLEONS

→ 'INERT' CORE AND A NUMBER OF VALENCE NUCLEONS

→ CONSTRUCT A 'BASIS' IN THE VALENCE SPACE $\varphi_{J,i}$

$$\varphi_{J,i} = \{(j_1)^{n1}_{J_1} \ (j_2)^{n2}_{J_2} \ldots \}_J$$

AND SOLVE EIGENVALUE EQUATION

$$\hat{H}\psi_{J,k} = E_{J,k}\,\psi_{J,k}$$

$$\psi_{J,k} = \sum_i a^{k,i}_J\,\varphi_{J,i}$$

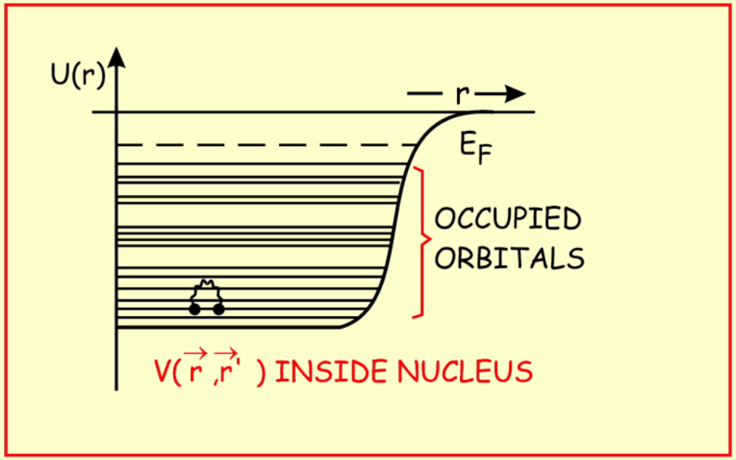
PARAMETERS:

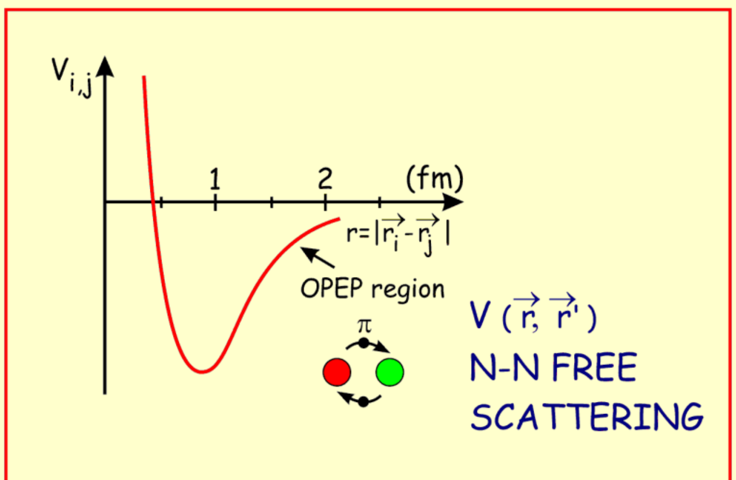
- SINGLE-PARTICLE ENERGIES ε_j
- TWO-BODY n-n INTERACTION $\langle j_1 j_2, J | V_{1,2} | j_3 j_4, J\rangle$

→ MANY STRUCTURES ARISE

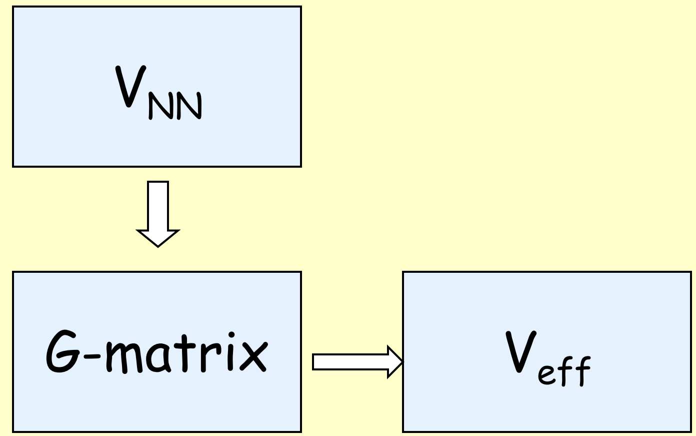
Microscopic effective interaction

A bare NN-potential - CD-Bonn, Nijmegen II, AV18, chiral N3LO potential - requires regularization and modification to be applied for many-nucleon systems in a restricted model space.





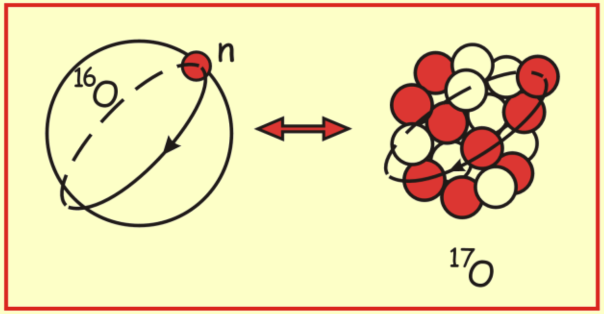
Conventional procedure from '60s to nowadays



expansion of effective interaction in terms of the nuclear reaction matrix G

M. Hjorth-Jensen et al, Phys.Rep.261 (1995)

Concept of effective interaction (operators) active in finite space:



$$\Psi = \sum_i a_i \Psi_i \{\text{17-nucleon coordinates}\}.$$

$$\hat{O} = \sum_{i=1}^{17} \hat{O}\,(\vec{r}_i, \vec{\sigma}_i, \vec{\tau}_i).$$

More general

$$(H_0 + V)\,\Psi = E\Psi \qquad \Psi = \sum_{i=1}^{\infty} a_i \Psi_i^{(0)}$$

FULL SPACE $(1,\ldots\infty)$

MODEL SPACE $(1,\ldots M)$ $\quad \Psi^M = \sum_{i=1}^{M} a_i \Psi_i^{(0)}$

IMPLICIT EQ. $\langle \Psi^M | H^{eff} | \Psi^M \rangle = E$

… still phenomenological adjustment required

Microscopic effective 2-body interactions (either G-matrix or V_{low-k}) fail to reproduce nuclear properties when the number of valence particles increases: the monopole part of the interaction if deficient (lack of 3-body forces)
$\Rightarrow$ phenomenological adjustment to data

E.Caurier et al, Rev. Mod. Phys. 77 (2005) 427

- Monopole part of the interaction adjusted (KB3, KB3G for pf-shell)

A.Poves, A.P.Zuker, Phys. Rep. 70 (1981)
G. Martinez-Pinedo et al, Phys. Rev. C55 (1997)

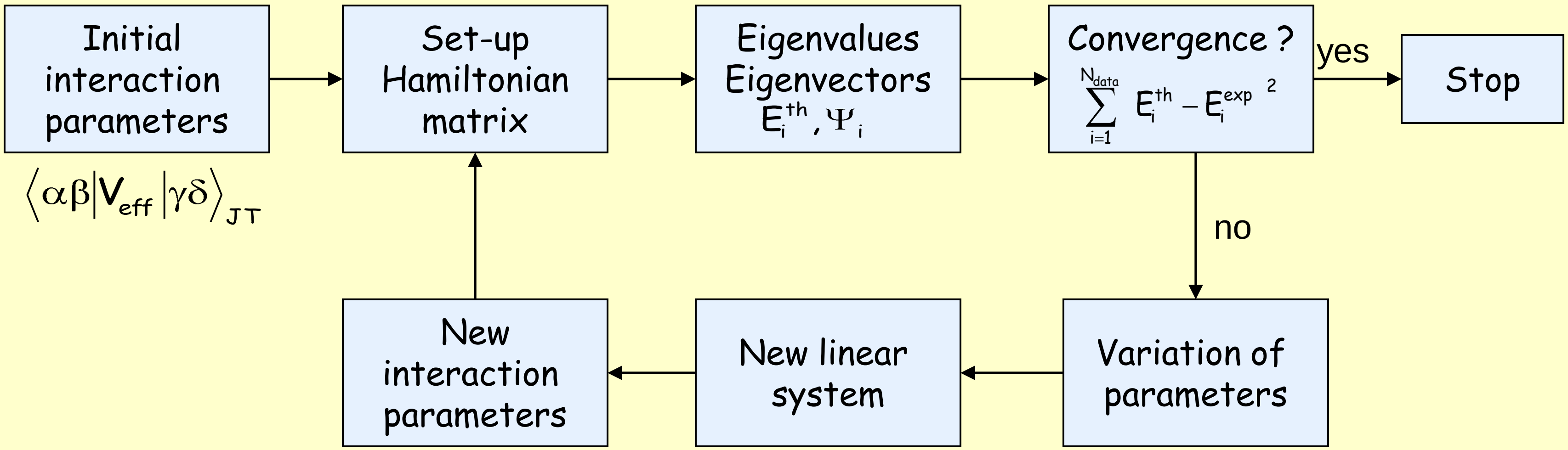
- Least-square fit of all the m.e. – by a linear-combination method (GXPF1 for pf-shell)

B.A.Brown, W.A.Richter, Phys. Rev. C74 (2006)
M. Honma et al, Phys. Rev. C65 (2002); idem 69 (2004)

If the model space contains all important degrees of freedom, the shell model is extremely powerful !

Empirical V_{eff} (least-square-fit method)

All two-body matrix elements (TBME) between valence nucleons in a model space are considered as free parameters.

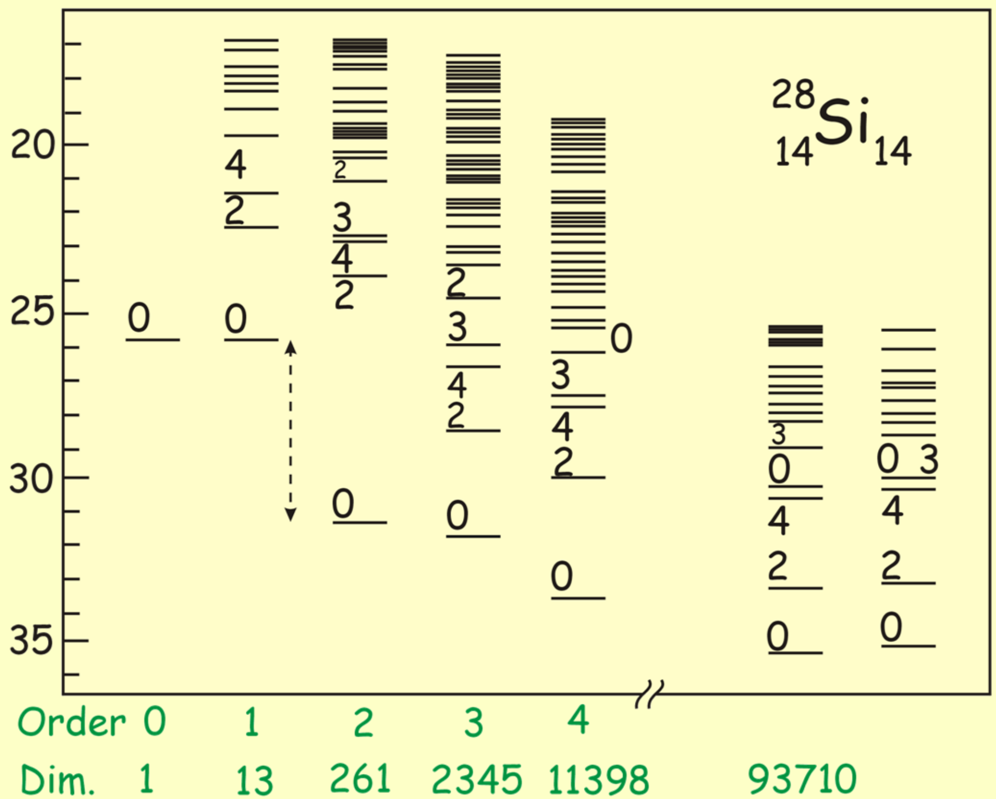


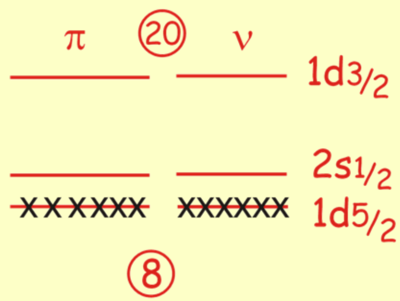
0p-shell: ^{4}He – ^{16}O 15 TBME
1s0d-shell: ^{16}O – ^{40}Ca 63 TBME
1p0f-shell: ^{40}Ca – ^{80}Zr 195 TBME

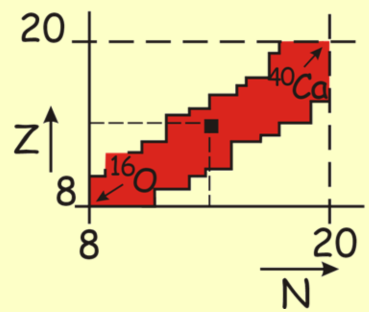
Cohen, Kurath (1965)
Brown, Wildenthal, USD (1988)
Tokyo-MSU, GXPF1 (2002,2004)

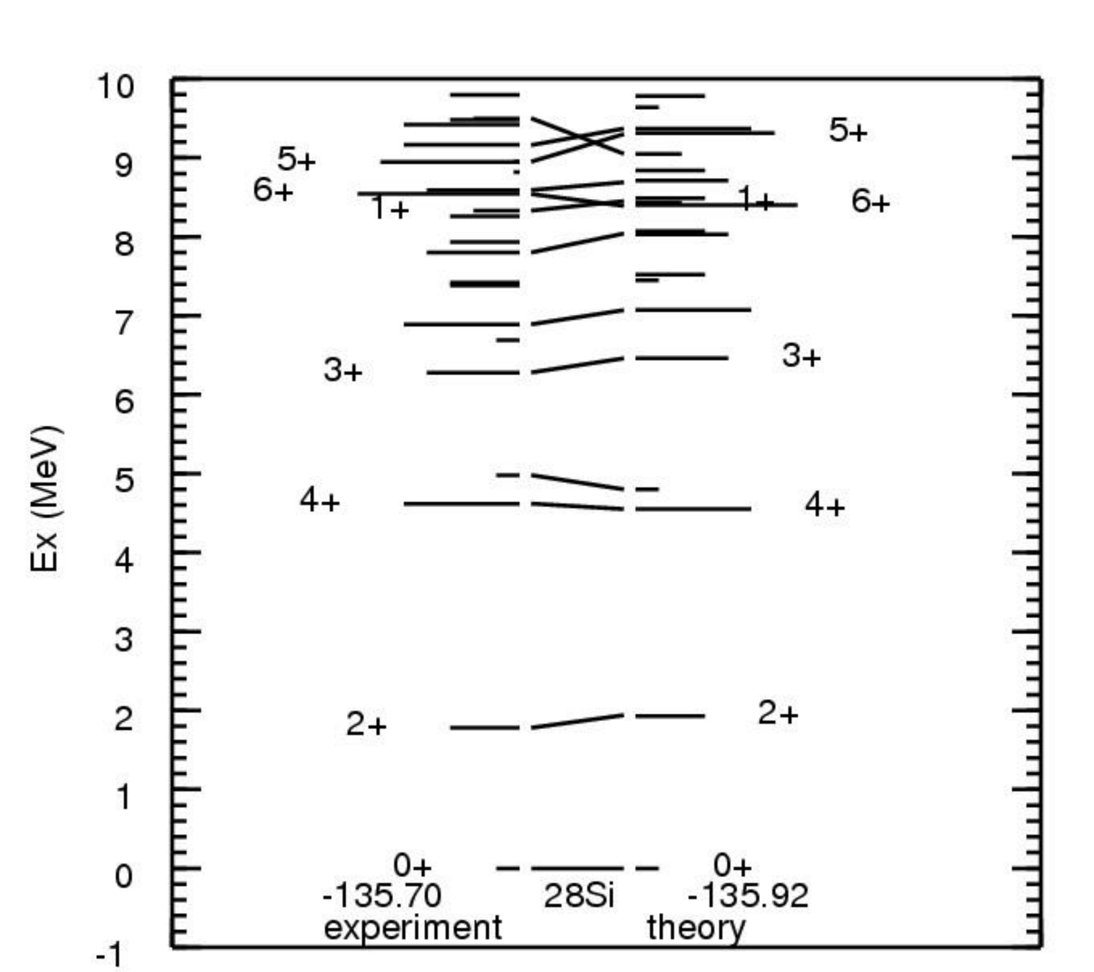
} *Linear combination method*

sd MODEL SPACE









^{28}Si using the USD-A Hamiltonian for the sd shell

B.A.Brown and W.A.Richter, Phys.Rev.C74(2006),034315

http://www.nscl.msu.edu/~brown/resources/usd-05ajpg/si28.jpg

* VERY - LIGHT NUCLEI ($A \lesssim 12$): AB - INITIO CALCULATIONS

* SHELL - MODEL

1p shell : Cohen, Kurath (1965): 15 2b m.e.
2s, 1d shell : Brown, Wildenthal (OXBASH) (1988, 2007): 63 2b m.e.
2p, 1f shell : Hjorth-Jensen, Kuo, Osnes (1995): Bonn (C) potential (1996)
Madrid - Strasbourg, (ANTOINE) (Kuo, Brown-KB force (1996))
KB (fp), KB3, KB3G,...
Honma, Otsuka, Brown, Mizusaki: GXPF1,...(2004)
} 195 2b m. e.
ex. ^{56}Ni: full fp shell: 10^9 (all M states) (2007)
2p, 1f, $1g_{9/2}$: ? without reach 2.4×10^{20}(all M states)

* RESTRICTIONS

$C^{n_\pi}_{N_\pi} \cdot C^{n_\nu}_{N_\nu}$

n_π (n_ν) : number of active protons (neutrons)
N_π (N_ν) : number of single-particle (j,m) states for protons (neutrons)

Schematic interaction

Some examples

$$V(1,2) = -V_0 \frac{e^{-\mu r}}{\mu r}$$ Yukawa potential

$$V(1,2) = -V_0\, \delta(\vec{r}_1 - \vec{r}_2)$$

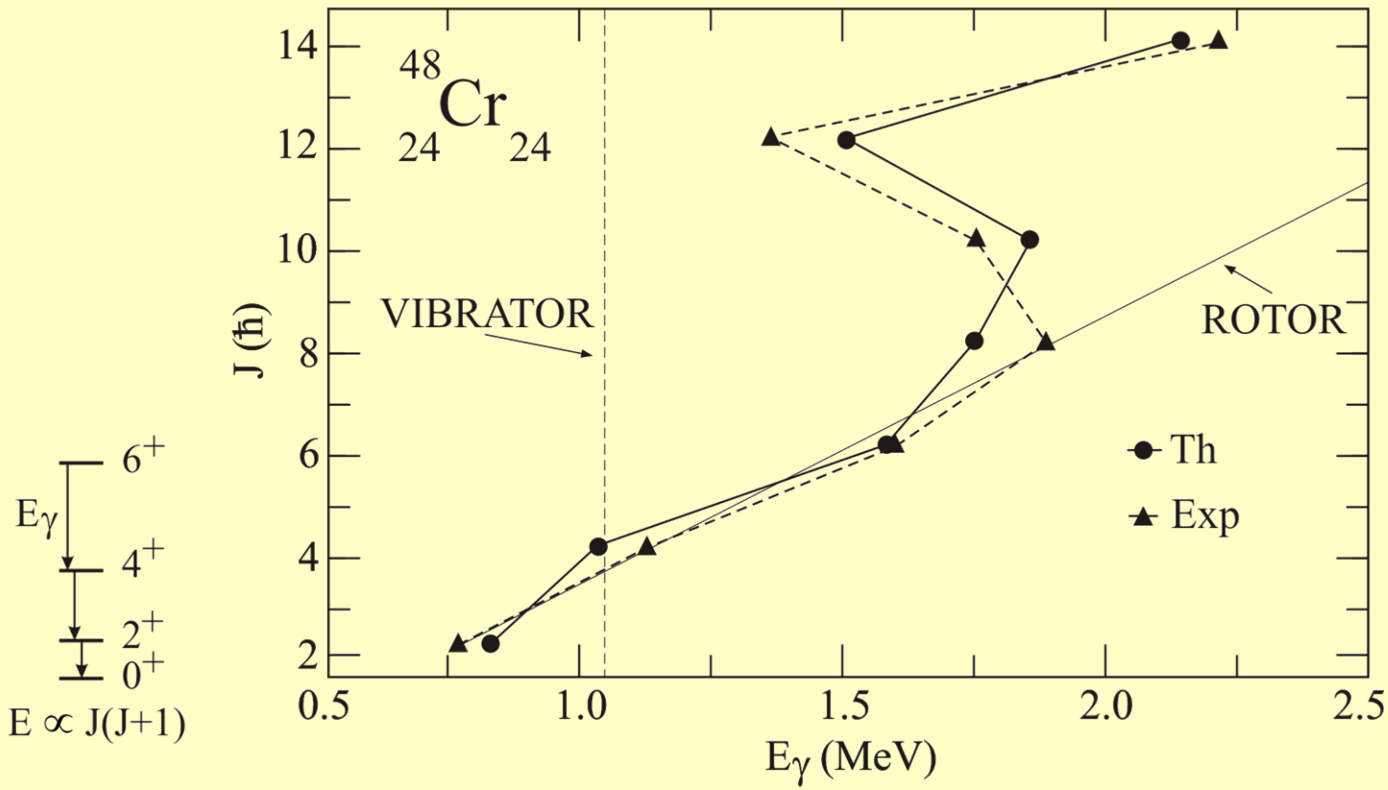
$$V(1,2) = -V_0\, \delta(\vec{r}_1 - \vec{r}_2)\,(1+\alpha\vec{\sigma}_1 . \vec{\sigma}_2)$$

(both) δ-forces

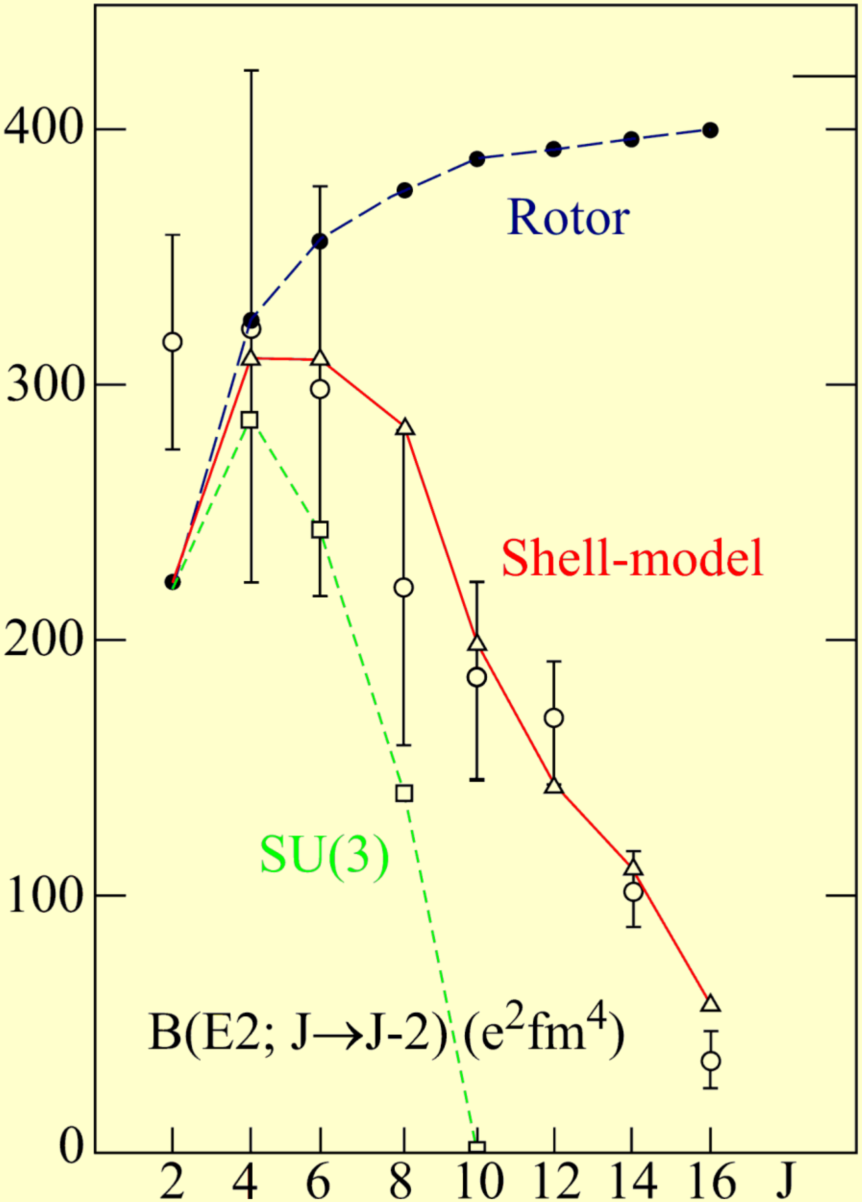
$$V(1,2) = -V_0\, \delta(\vec{r}_1 - \vec{r}_2)\, \delta(r_1 - R)$$ Surface-δ interaction (SDI)

$$V(1,2) = \chi\, \hat{Q} . \hat{Q} \quad (\text{with } Q_\mu = r^2 Y_{2\mu}(\Omega_r))$$ Quadrupole-quadrupole interaction

STATE-OF-THE ART SHELL-MODEL CALCULATION IN fp SHELL



Strasbourg-Madrid
(CAURIER, NOWACKI, ZUKER, POVES et al.)



Rotational model:

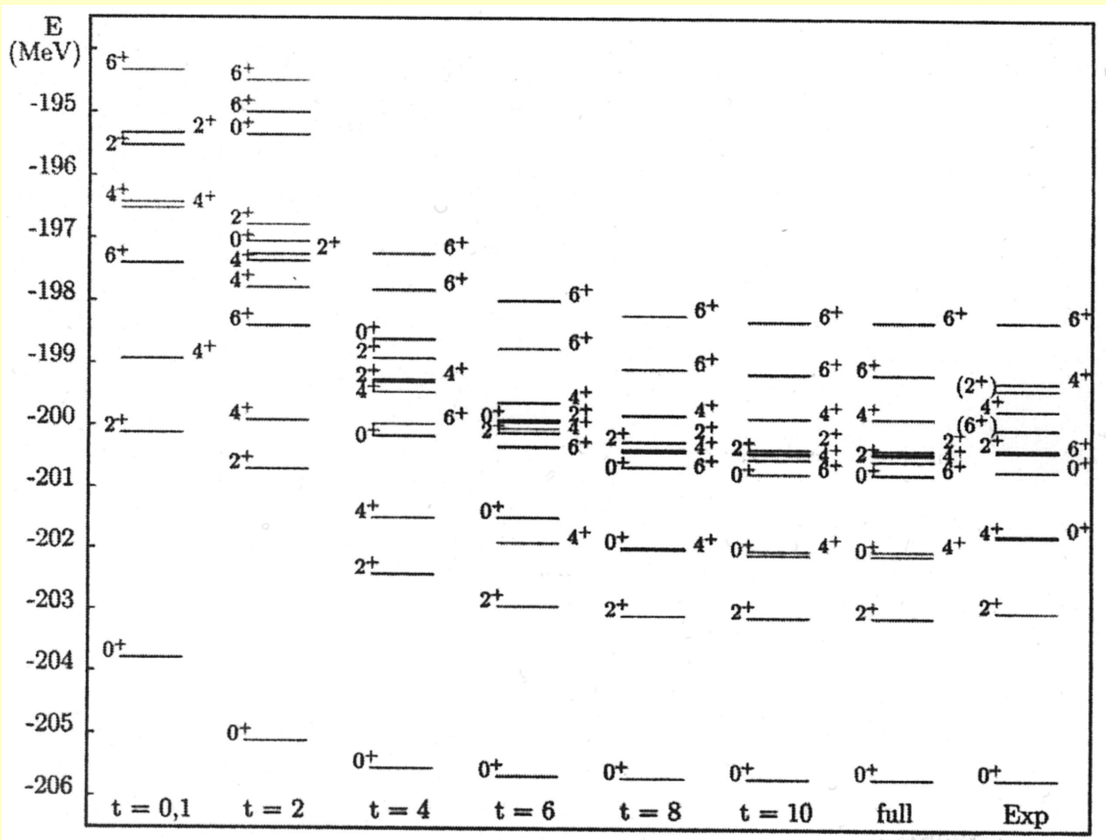
$$B(E2;\, J\rightarrow J-2) = \frac{5}{16\pi}\, e^2\, \frac{1}{4}\, \frac{J(J-1)}{(2J-1)(2J+1)}\, Q_0(t)$$

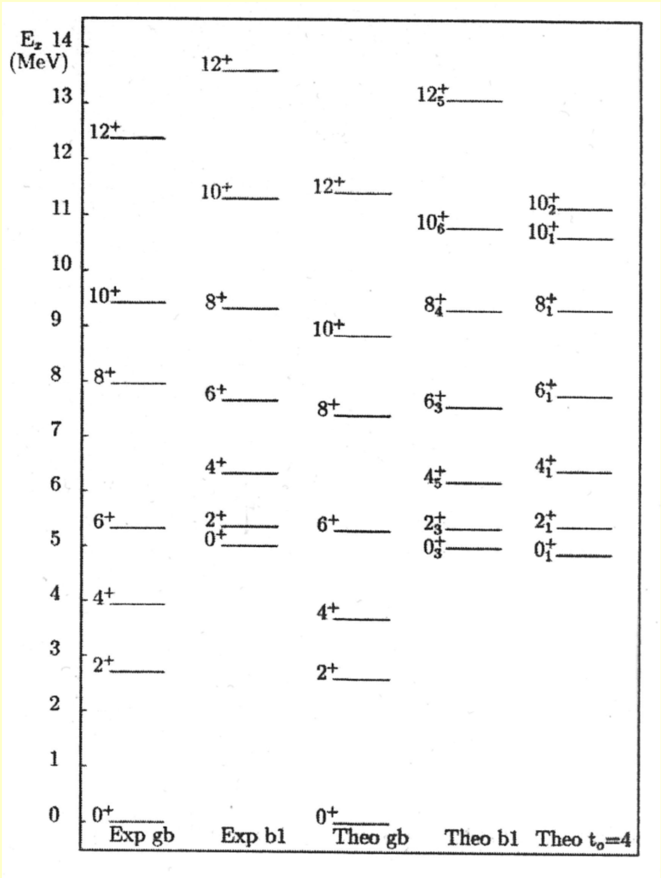
$Q_0(t)$:"intrinsic" property-constant for pure rotor.

Extract $Q_0(t)$ from experiment

J = 2	107
4	105
6	100
8	93
10	77
12	65
14	55
16	40

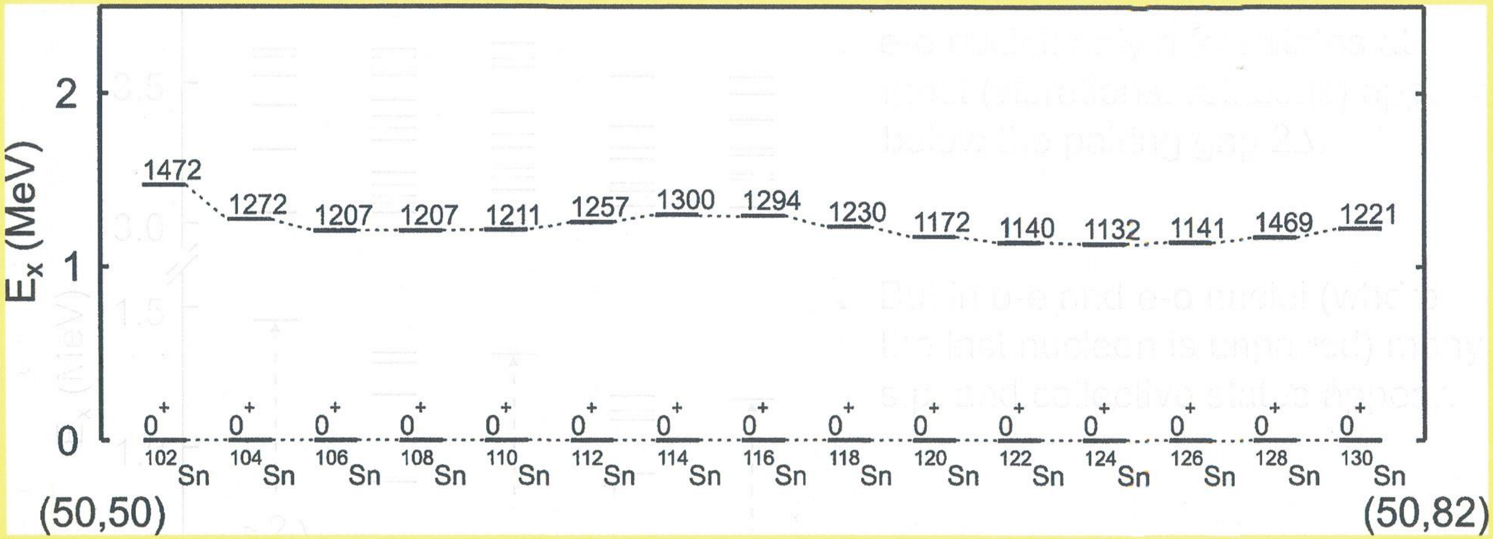
Full fp shell-model study of ^{56}Ni (Horoi et al., PRC 73(2006), 061305)





$(1f_{7/2})^{16-t}(2p_{3/2}\,1f_{5/2}\,2p_{1/2})^{t}$

(iii) The excitation energy of the first excited 2^+ state in nuclei remains remarkably constant over large intervals of neutron (proton) numbers.
Example: 2_1^+ excitation energy in Sn nuclei



- These 2^+ states are not rotational states but are connected to a coherent pairing condensate.
- Pair breaking energy: $2\Delta \approx 2$ MeV

Extreme pairing model

$S_j^+|0\rangle = |j^2, J=0, M=0\rangle$ state

Basis $(S_j^+)^{\frac{n}{2}}|0\rangle$,

$(S_j^+)^{\frac{n}{2}-1}(B_j^+)|0\rangle, \ldots$

$\hat{H} = -G S_j^+ S_j \; (j+\frac{1}{2})$

$\Downarrow$

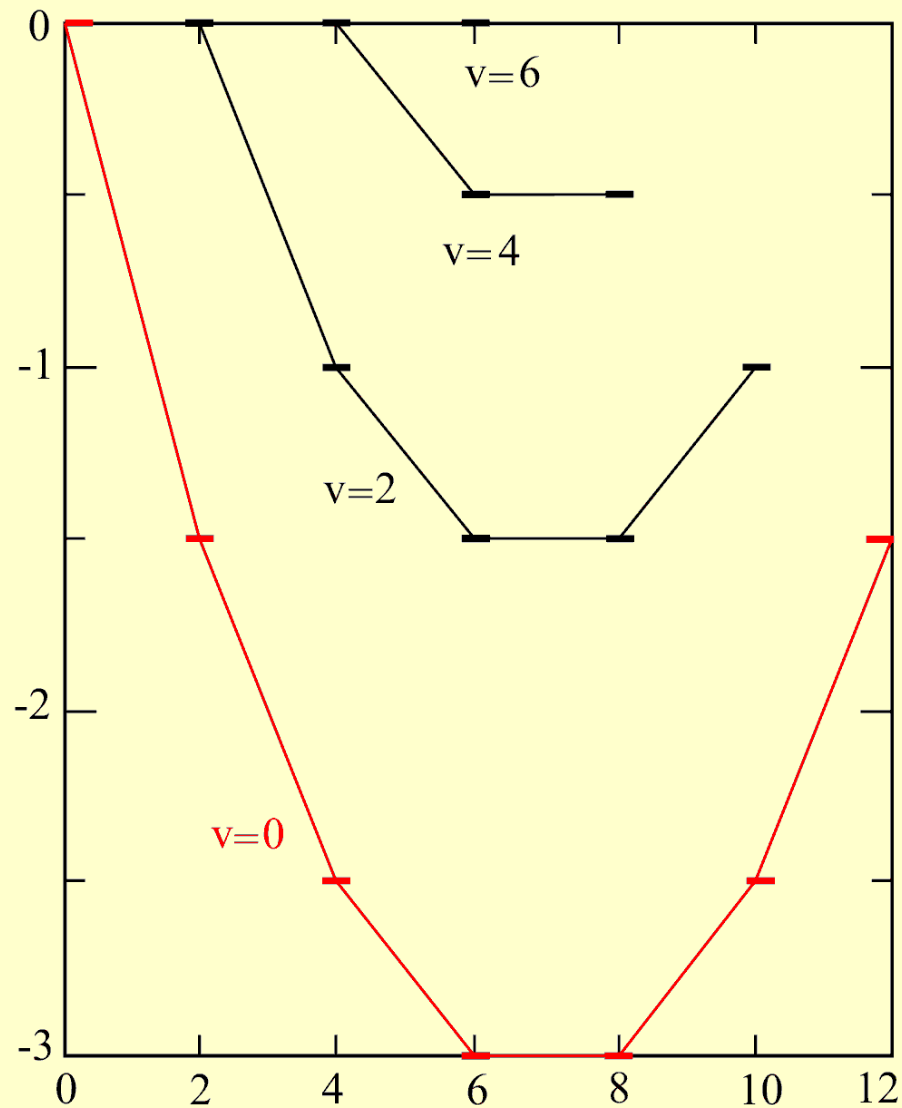
Exactly solvable

$\Downarrow$

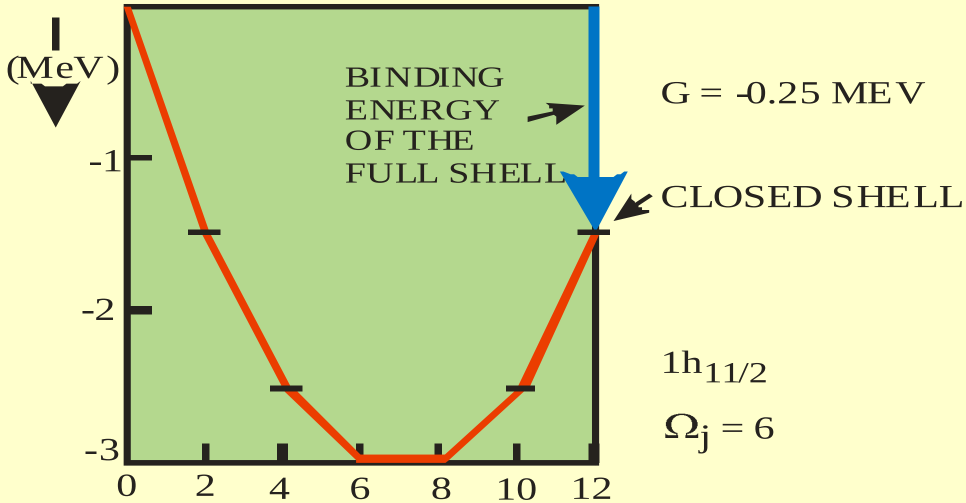
$E(v,n) = \frac{-G}{4}(n-v)(2\Omega_j - n - v + 2)$

v: 2x number of "broken" pairs.

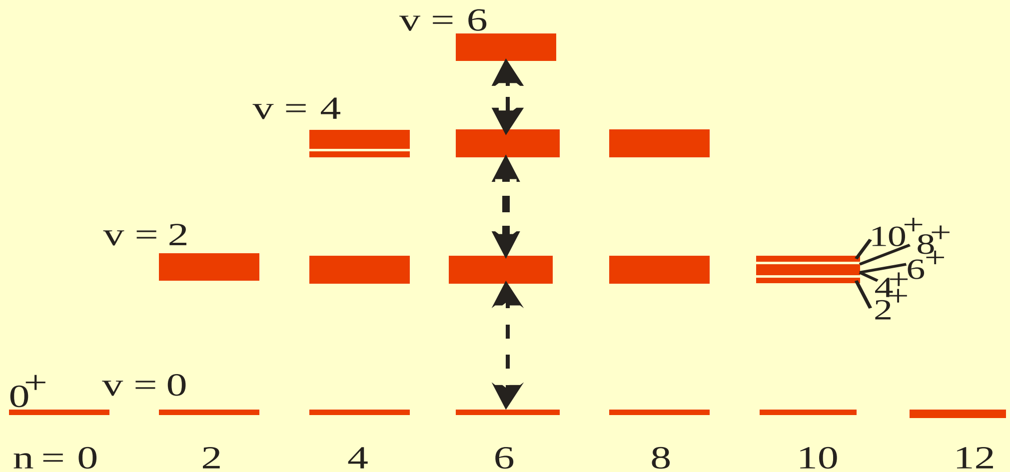
$1h_{11/2}$ orbital

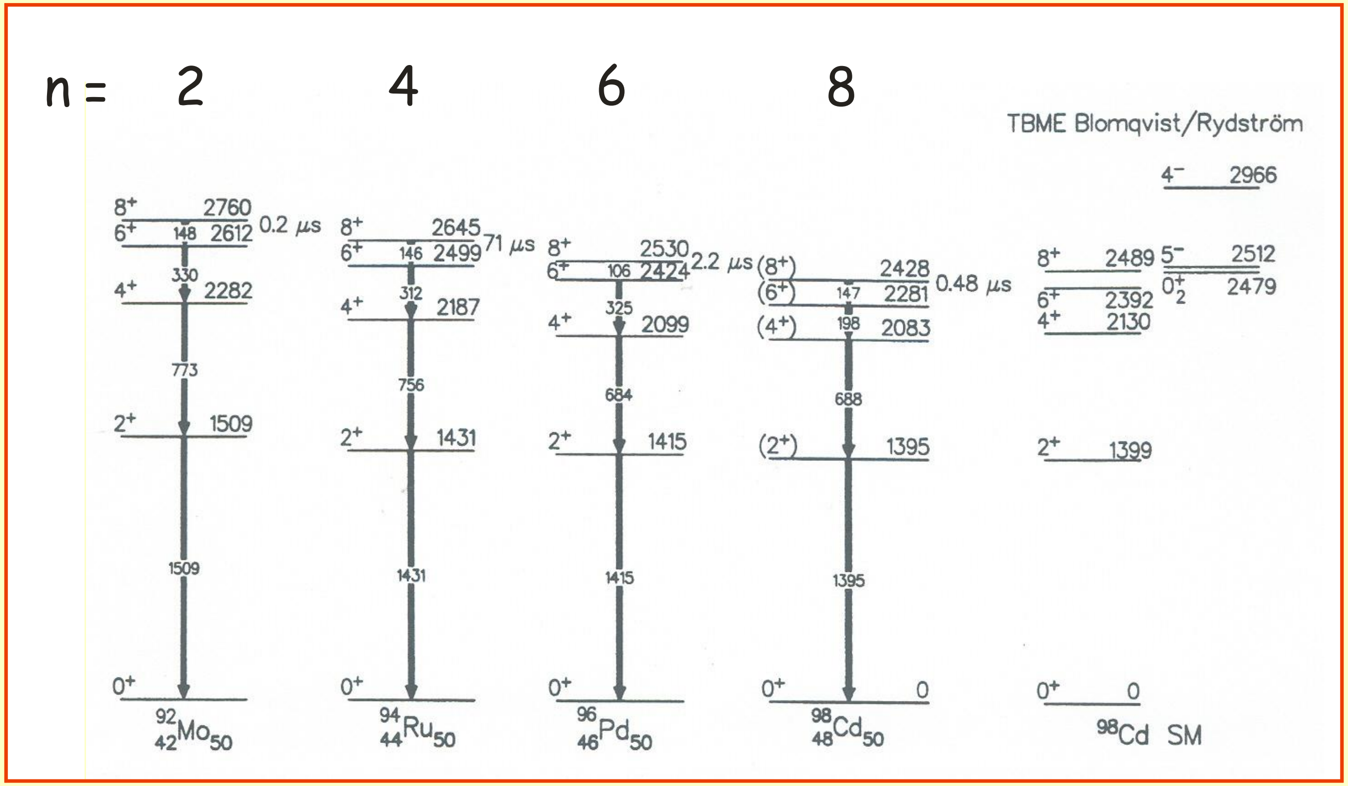


MANY - BODY (IDENTICAL) CORRELATIONS



$$\hat{H}\,|n,v=0\rangle = -\frac{G}{4}\,n\,(2\Omega j - n + 2)\,|n,v=0\rangle$$





N=50 nuclei

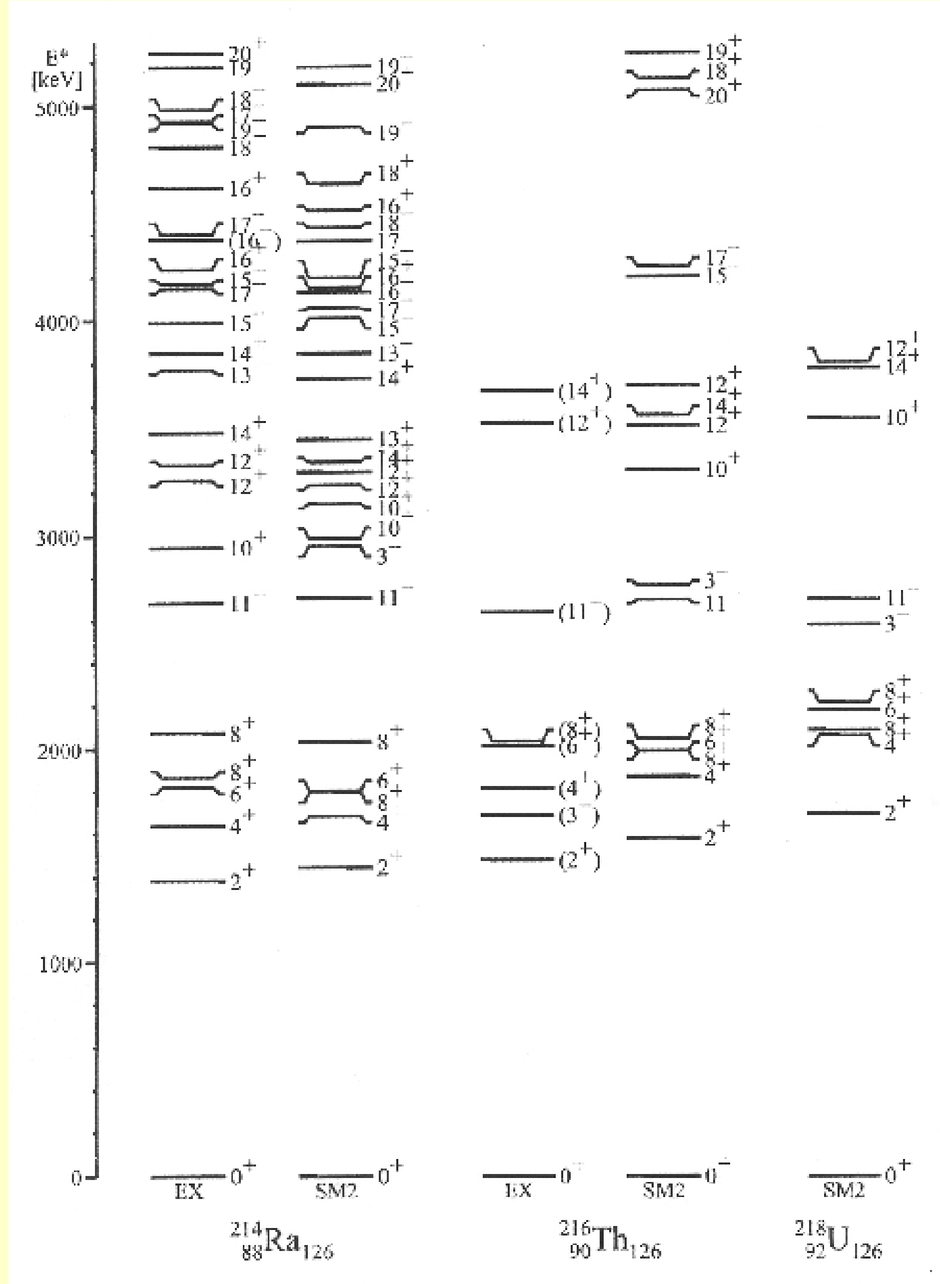
Filling the $1g_{9/2}$ orbital

$(1g_{9/2})^{-2}$

CONFIGURATION

$(2p_{1/2})^{-2}$

$(2p_{1/2}^{-1}\,1g_{9/2})$



Shell-model and experimental level schemes for the even N=126 isotones ^{214}Ra, ^{216}Th and ^{218}U.

E. Caurier et al., PRC 67, 054310 (2003)

Summary and Conclusions

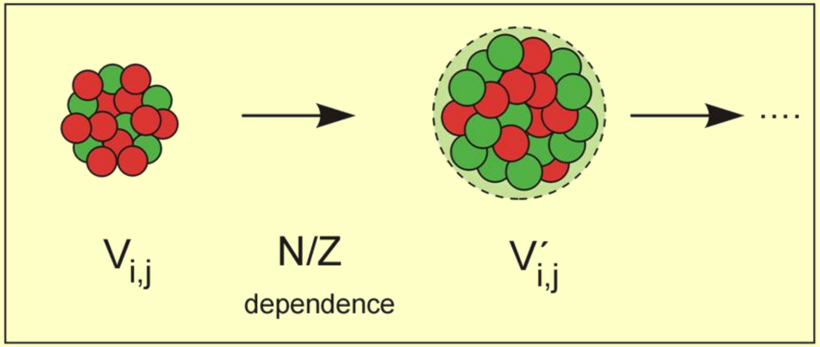
- Shell model represent a powerful theoretical model to describe low-energy nuclear spectroscopy
- Having got $E_{J,k}$, $\Psi_{J,k}$ one can calculate matrix elements of operators to compare with experiment (spectroscopic factors, static and transition electromagnetic moments - Q, μ, B(E2), …, weak decays - β, $\beta\beta$, lifetimes, etc)
- There is a clear link to the NN interaction, although more developments in the effective interaction theory is required

The shell model as unified view of nuclear structure
E.Caurier et al, Rev. Mod. Phys. 77 (2005) 427

NOT AT ALL SURE THESE "CONCEPTS" WILL ALL REMAIN VALID MOVING AWAY FROM STABILITY (Sp=0 Sn=0)

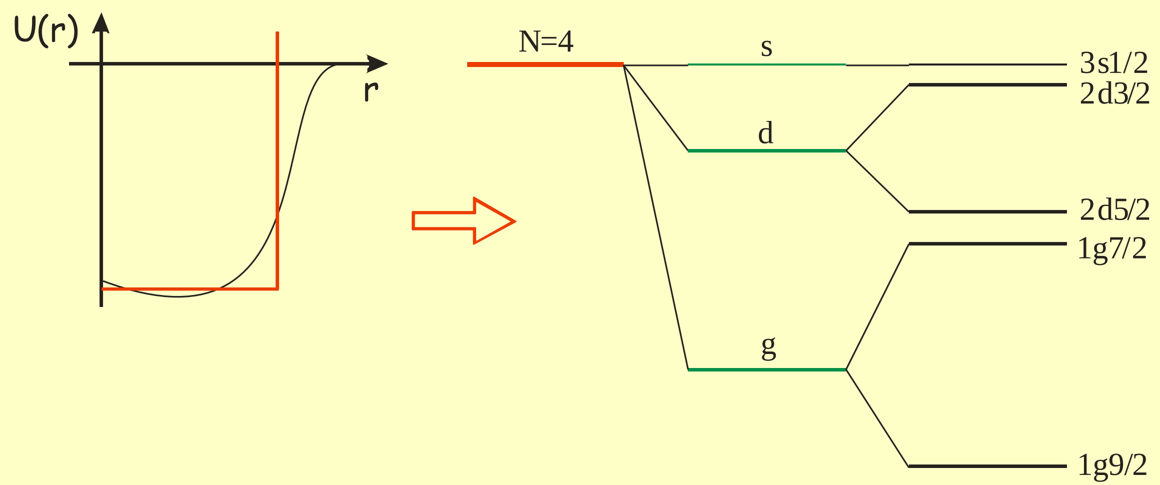
EXTRAPOLATIONS ? VERY DANGEROUS

MOST METHODS (SHELL-MODEL, COLLECTIVE MODELS, MEAN-FIELD STRUCTURE) HAVE A CENTRAL BIAS i.e. DEVISED FROM REGION AT AND NEAR STABILITY

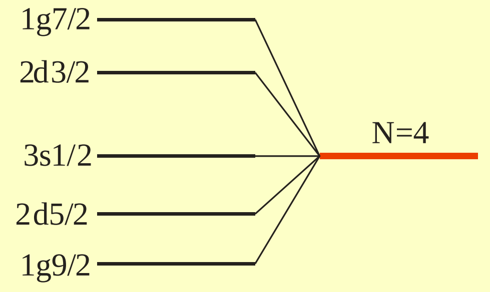


USE "ROBUSTNESS" OF SHELL-MODEL TO EXPLORE LIMITS-COUPLING WITH MEAN-FIELD PROPERTIES e.g. $\varepsilon_{s.p}$

B. Changing mean-field and 'islands' of deformation

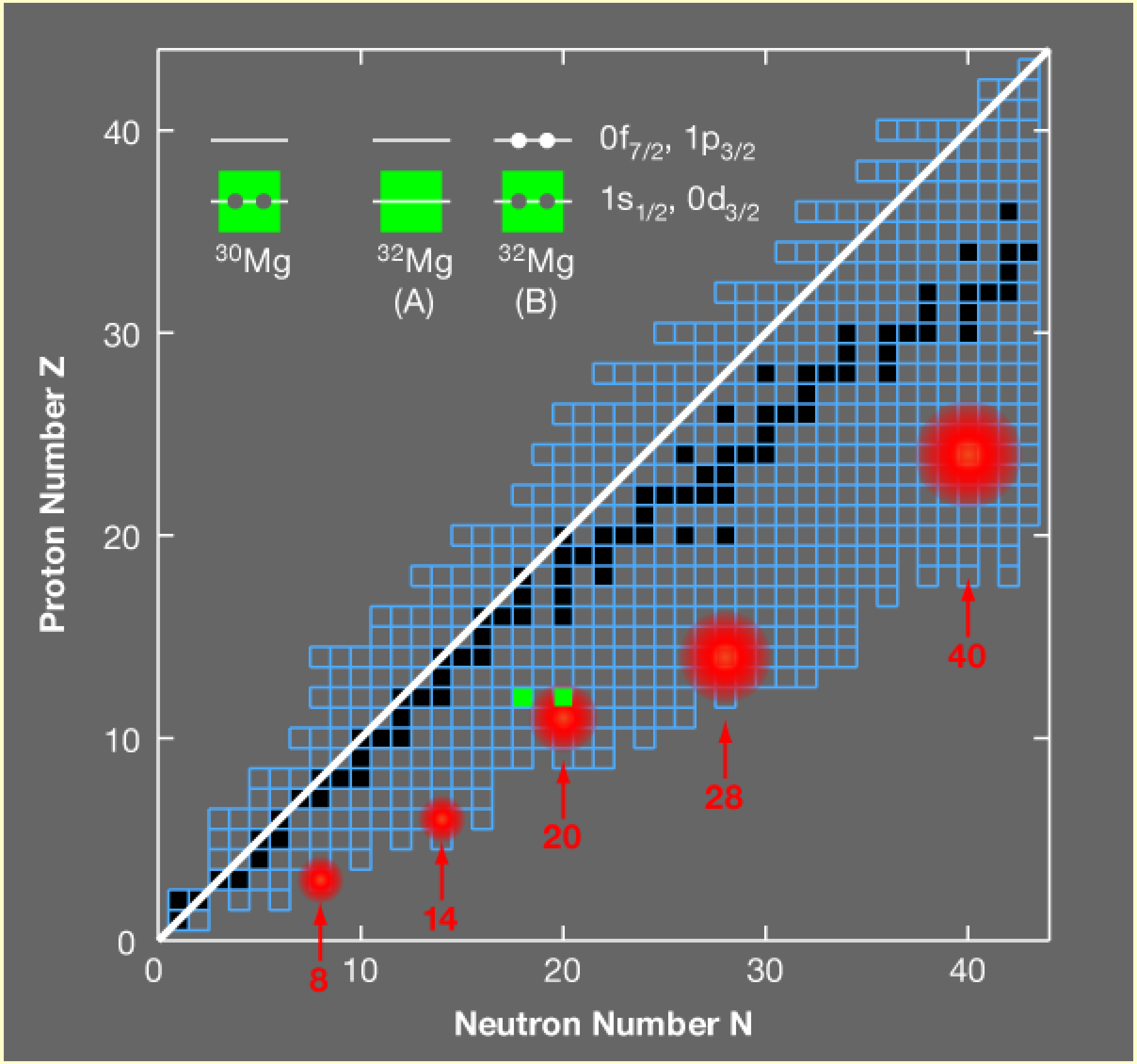


Potential will change far from region of β-stability (diffuseness,...)



HF: $U(\vec{r}) = \int \rho(\vec{r}')\, V(\vec{r}, \vec{r}')\, d\vec{r}'$

ε_{nlj} $\qquad \rho(\vec{r}') = \sum_{b \varepsilon F} |\varphi_b(\vec{r}')|^2$



$0f_{7/2}$, $1p_{3/2}$
$1s_{1/2}$, $0d_{3/2}$
^{30}Mg
^{32}Mg (A)
^{32}Mg (B)
Proton Number Z
Neutron Number N
0
10
20
30
40
8
14
20
28
40

References

Shell-model theory

Shell-Model Applications in Nuclear Spectroscopy
P.J.Brussaard, P.W.M.Glaudemans
North-Holland (1977)

The Nuclear Shell Model, K.Heyde
Springer-Verlag (1994)

The shell model (Le modèle en couches), A.Poves
Ecole Internationale Joliot-Curie (1997)

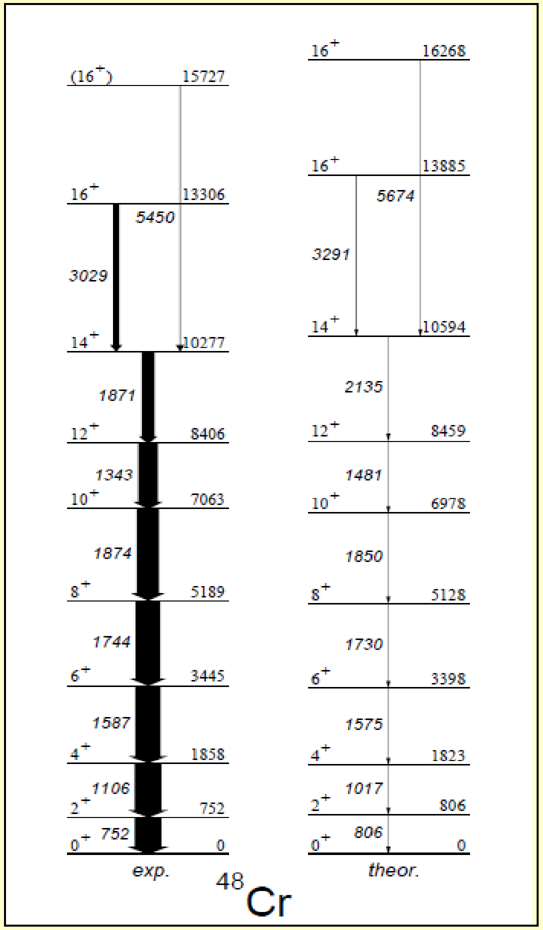
The shell model as unified view of nuclear structure
E.Caurier et al, Rev. Mod. Phys. 77 (2005) 427

Effective interactions

B.H.Brandow, Rev. Mod. Phys. 39 (1967) 771
B.Barrett, M.W.Kirson, Adv. Nucl. Phys. 6(1974) 219

M. Hjorth-Jensen, T.T.S.Kuo, E.Osnes, Phys. Rep. 261 (1995) 125
D.Dean et al, Prog. Part. Nucl. Phys. 53 (2004) 419
S.Bogner , T.Kuo, A.Schwenk, Phys. Rep. 386 (2003) 1

^{48}Cr in *pf*-shell model space



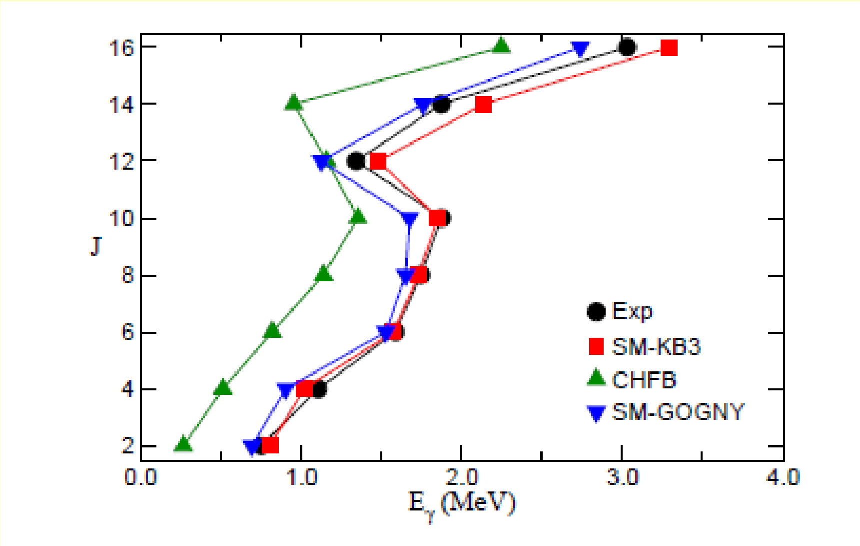
KB3 (semi-empirical interaction in *pf*-shell model space)
Strasbourg-Madrid

For J<10 :

$$E_J \sim J(J+1)$$

$$Q_0 = \frac{(J+1)(2J+3)}{3K^2 - J(J+1)} Q_{spec}(J), \; K \neq 1$$

$$B(E2; J \rightarrow J-2) = \frac{5}{16\pi} e^2 \left| \langle JK20 \,|\, J-2, K \rangle \right|^2 Q_0^2$$



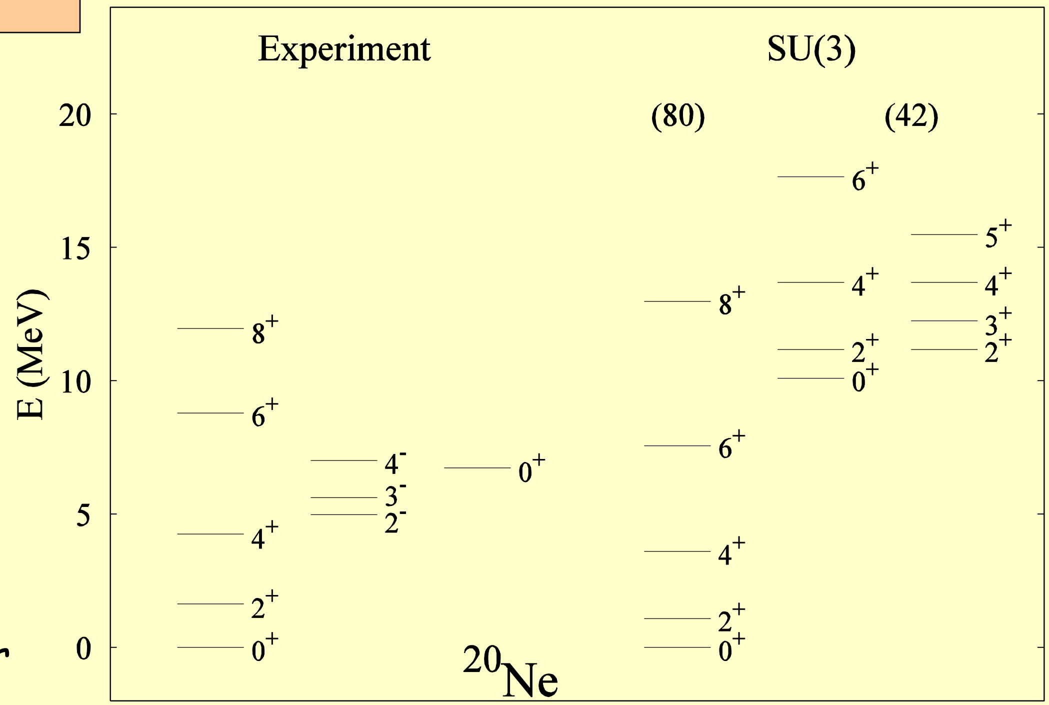
J<10: collective rotation
J=10-12 : backbending phenomenon (competition between rotation and alignment of $0f_{7/2}$ particles)
J>12 : spherical states

Figures from E.Caurier et al, Rev. Mod. Phys. 77 (2005) 427

^{20}Ne (Z=10,N=10) and SU(3) model of Elliott

$$H = \sum_{k=1}^{A} \left[\frac{p_k^2}{2m} + \frac{1}{2} m\omega^2 r_k^2 \right] - \chi Q \cdot Q$$

J.P.Elliott (1958)



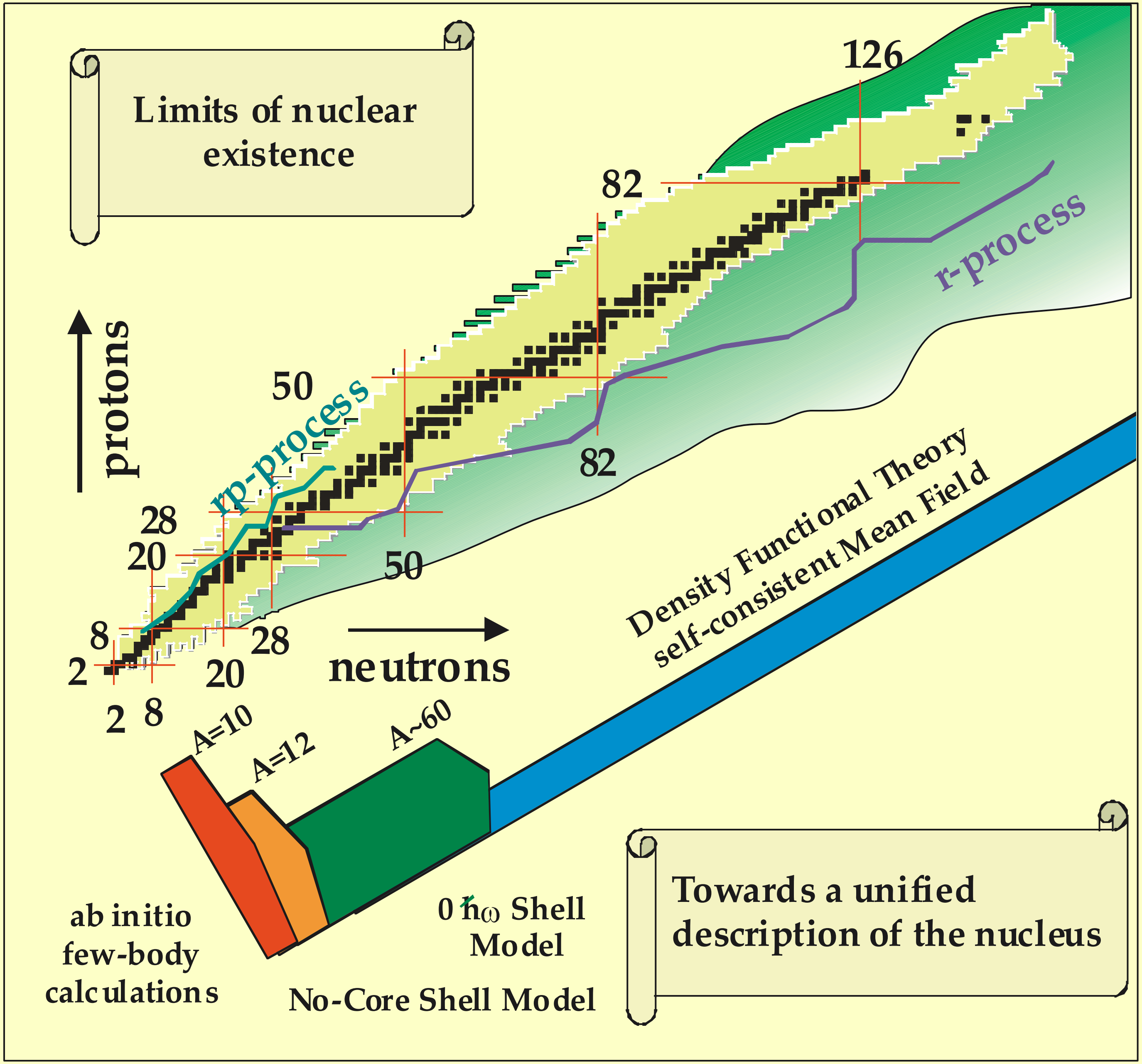
Rotational classification of nuclear states (mixing of many spherical configurations)

Q is an algebraic quadrupole operator

$$Q_\mu = \sqrt{\frac{4\pi}{5}} \left(\sum_k r_k^2 Y_{2\mu}\ \ \Omega_r\ \ / b^2 + b^2 \sum_k p_k^2 Y_{2\mu}\ \ \Omega_p\ \ / \hbar^2 \right)$$

$$L_\mu = \sum_k \ r_k \times p_k\ \ _\mu / \hbar$$

SU(3) generators


Limits of nuclear existence
126
82
50
rp-process
r-process
protons
neutrons
28
20
8
2
2
8
20
28
50
82
Density Functional Theory
self-consistent Mean Field
A=10
A=12
A~60
ab initio few-body calculations
0 ħω Shell Model
No-Core Shell Model
Towards a unified description of the nucleus

- Add more particles 2, 3, ..n one can use $n \rightarrow n+1 \rightarrow n+2$ coupling (c.f.p:coeff.of fractional parentage)

$$\psi(j^n \alpha; JM) = \sum_{\alpha', J'} [j^{n-1}(\alpha' J')j| \} j^n \alpha J] \, \psi(j^{n-1}(\alpha' J')j; JM)$$

Alternative method: construct Slater determinant (M) and project out J values

$$\psi(1,2,..A) = \frac{1}{\sqrt{A!}} \begin{bmatrix} \varphi_{\alpha_1}(\vec{r}_1) & \varphi_{\alpha_1}(\vec{r}_2) & \dots & \varphi_{\alpha 1}(\vec{r}_A) \\ \varphi_{\alpha_2}(\vec{r}_1) & \varphi_{\alpha_2}(\vec{r}_2) & \dots & \varphi_{\alpha 2}(\vec{r}_A) \\ \vdots & & & \\ \varphi_{\alpha_A}(\vec{r}_1) & \varphi_{\alpha_A}(\vec{r}_2) & \dots & \varphi_{\alpha A}(\vec{r}_A) \end{bmatrix}$$

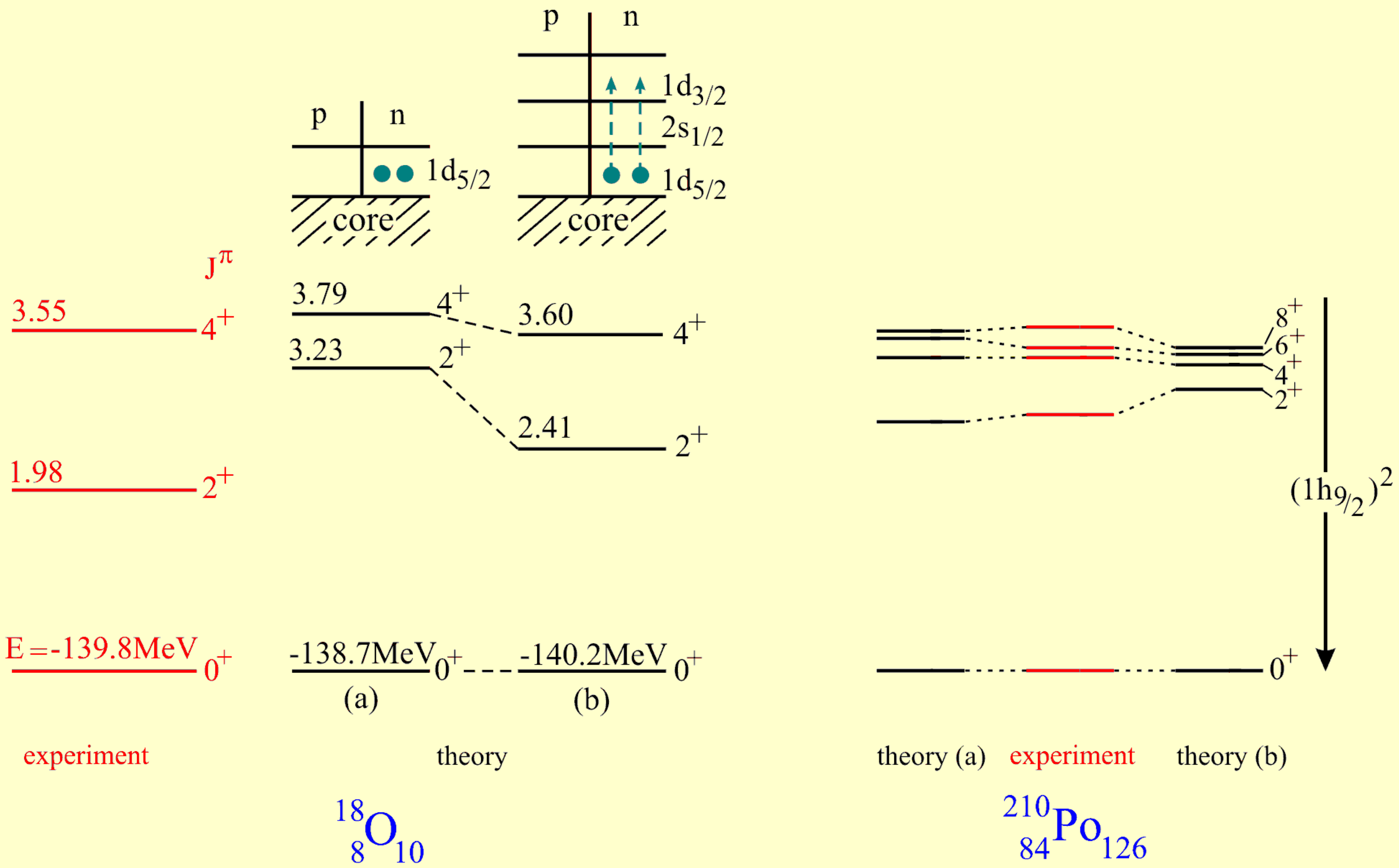
↓

A very convient way for computing

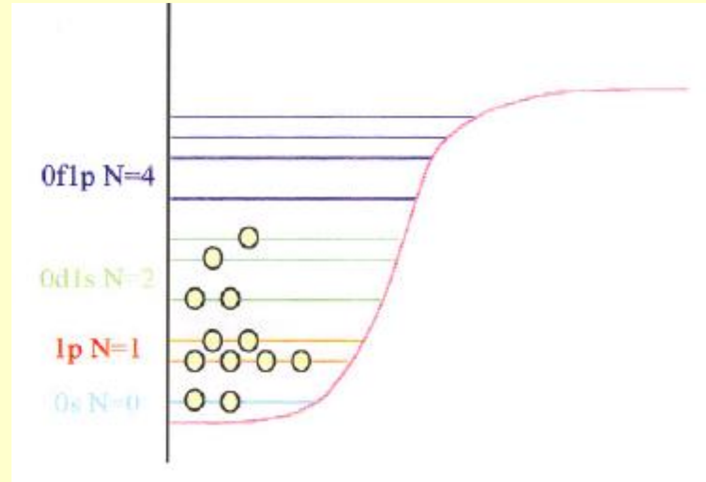
Two major questions

- How to build a shell-model basis in an optimal way (computational).
- How to handle the effective nucleon-nucleon interaction V_{eff}.

  - Schematic interaction (parameterized interaction in nuclear medium).
  - Empirical effective interaction (fitting ε_j, 2-body m.e. to nuclear properties such as E_x,B(E2), B(M1),μ,Q,β-decay) in a model space.
  - Microscopic (effective) interaction (derived from a realistic nucleon-nucleon force).



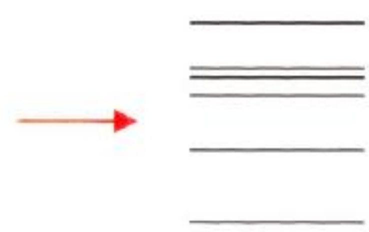
p
n
1d5/2
core
p
n
1d3/2
2s1/2
1d5/2
core
Jπ
3.55
4+
1.98
2+
E =-139.8MeV
0+
3.79
4+
3.23
2+
-138.7MeV
0+
(a)
3.60
4+
2.41
2+
-140.2MeV
0+
(b)
experiment
theory
8+
6+
4+
2+
0+
(1h9/2)2
theory (a)
experiment
theory (b)
18 8 O 10
210 84 Po 126



Calculate Hamiltonian matrix $H_{ij}=\langle\phi_j|H|\phi_i\rangle$

— Diagonalize to obtain eigenvalues

$$\begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1N} \\ H_{21} & H_{22} & & \\ \vdots & & \ddots & \\ H_{N1} & & \cdots & H_{NN} \end{pmatrix}$$



THE PRACTICAL SHELL MODEL

- Choose a model space to be used for a range of nuclei
  - E.g., the 1d and 2s orbits (sd-shell) for ^{16}O to ^{40}Ca or the 1f and 2p orbits for ^{40}Ca to ^{80}Zr
- We start from the premise that the effective interaction exists
- We use effective interaction theory to make a first approximation (G-matrix)
- Then tune specific matrix elements to reproduce known experimental levels
- With this empirical interaction, then extrapolate to all nuclei within the chosen model space

The empirical shell model works well!
But be careful to know the limitations!