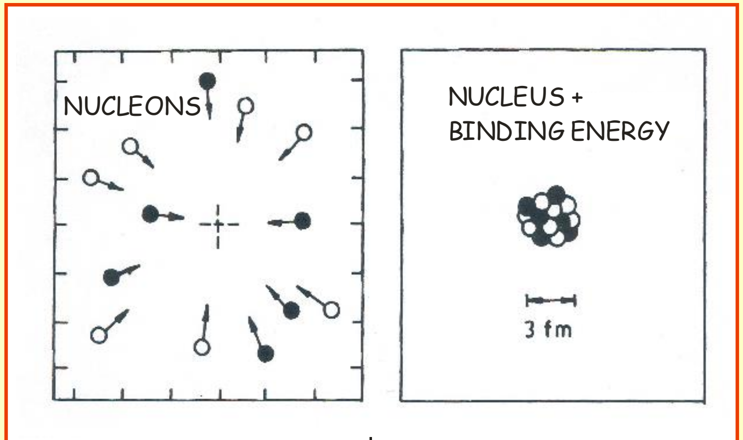
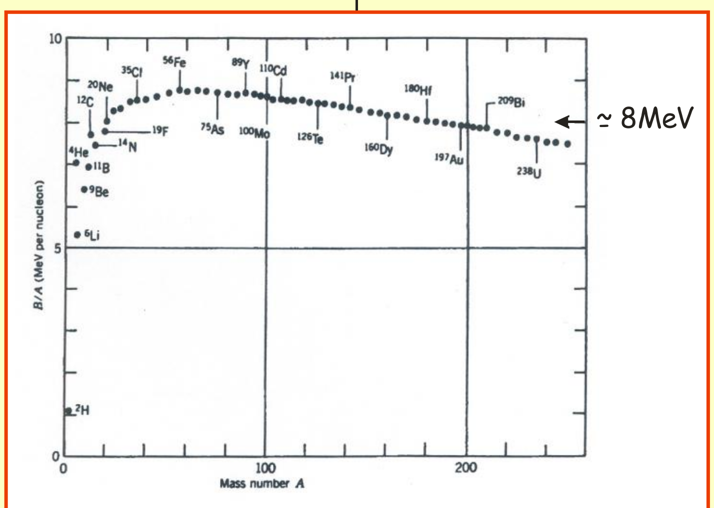
The nuclear shell-model: from single-particle motion to collective effects

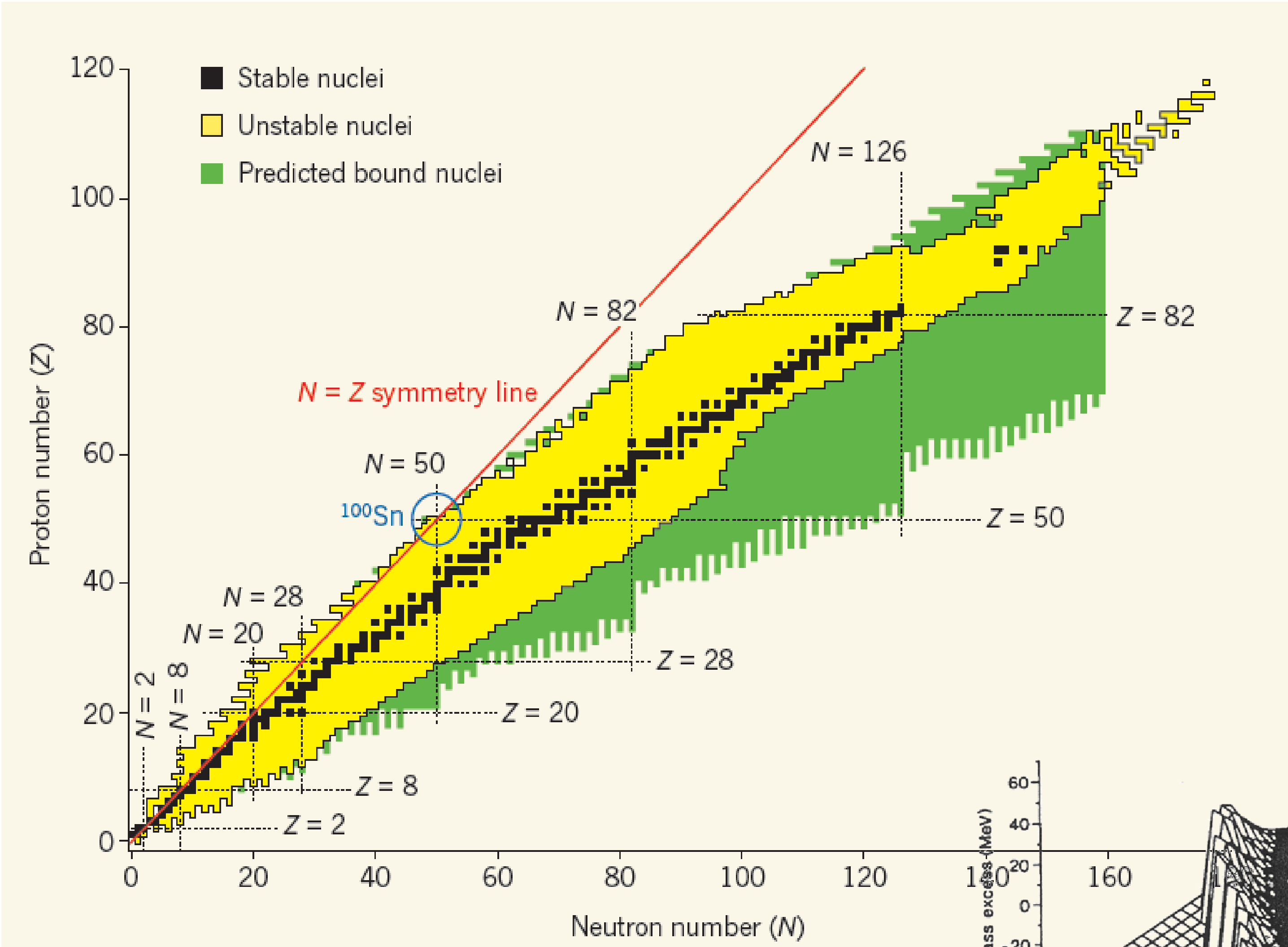
Some basic facts about the atomic nucleus

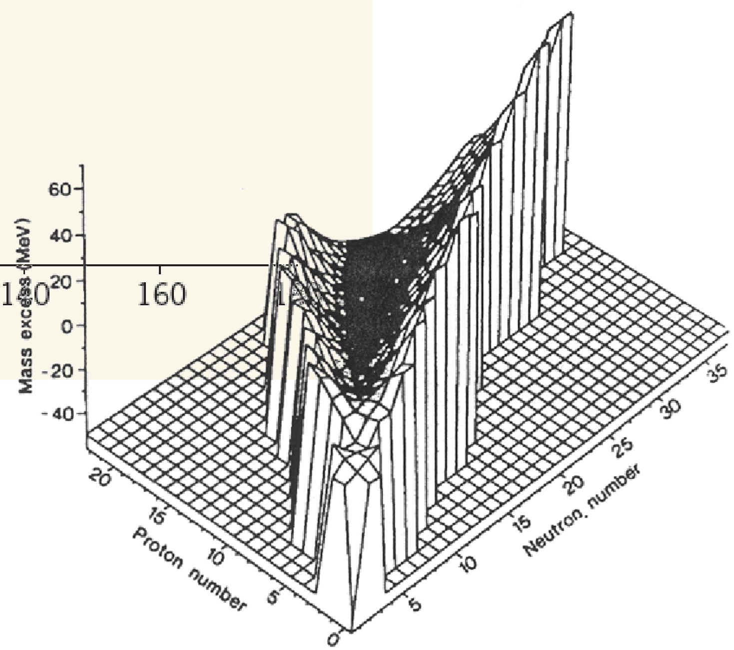
THE ATOMIC NUCLEUS AS A BOUND SYSTEM



NUCLEAR BINDING ENERGY / NUCLEON




Stable nuclei
Unstable nuclei
Predicted bound nuclei
N = Z symmetry line
^{100}Sn
N = 2
N = 8
N = 20
N = 28
N = 50
N = 82
N = 126
Z = 2
Z = 8
Z = 20
Z = 28
Z = 50
Z = 82
Proton number (Z)
Neutron number (N)
0
20
40
60
80
100
120
140
160


Mass excess (MeV)
60
40
20
0
-20
-40
Proton number
Neutron number
0
5
10
15
20
25
30
35

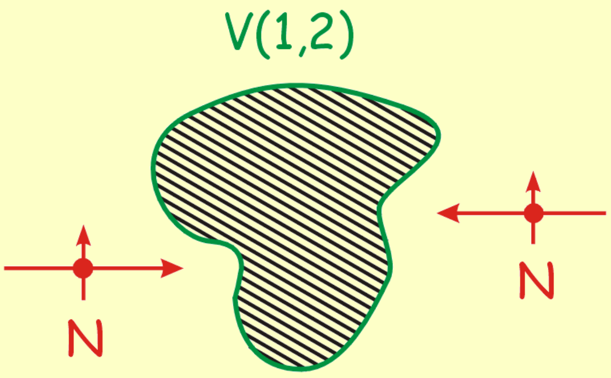
The nuclear shell-model: from single-particle motion to collective effects

1. Nuclear forces and very light nuclei

2. Independent-particle shell model and few nucleon correlations

3. Many-nucleon correlations: collective excitations and symmetries

REACTION CHANNELS



S=1, 0 : TRIPLET, SINGLET SPIN STATE

pp, pn and nn scattering states

ℓ: even, odd relative angular monentum states

➡ CHARACTERIZED BY $\delta(\ell, S)_{\mathcal{J}}$
PHASE SHIFTS IN SCATTERING PROCESS

$$\left[-\frac{\hbar^2}{2m_1}\Delta_1-\frac{\hbar^2}{2m_2}\Delta_2+V(1,2)\right]\psi(1,2)=E\,\psi(1,2)$$

⇩

SEPARATION IN RELATIVE + C.O.M. COÖRDINATES

$$\left[-\frac{\hbar^2}{2m_r}\Delta_r+V(r)\right]\psi(r)=E\,\psi(r)$$

$$\underset{r\to\infty}{\psi_k^+(r)}=e^{i\vec{k}.\vec{r}}+f(\theta,\varphi)\,\frac{e^{ikr}}{r}$$

⇩

PARTIAL WAVE EXPANSION

$$\sum_{\ell=0}^{\infty} i^{\ell}\,(2\ell+1)\,j_\ell(kr)P_\ell(\cos\theta)$$

$$j_\ell(kr)\;\xrightarrow{r\to\infty}\;\simeq\frac{1}{kr}\sin(kr-\ell\pi/2)$$

$$\cong i\left[\frac{e^{-i(kr-\ell\pi/2)}-e^{i(kr-\ell\pi/2)}}{2kr}\right]$$

FULL WAVE FUNCTION $(r \to \infty)$

$$\Rightarrow \frac{i}{2k} e^{-i\delta_\ell(k)} \left[\frac{e^{-i(kr-\ell\pi/2)}}{r} - S_\ell(k) \frac{e^{i(kr-\ell\pi/2)}}{r} \right]$$

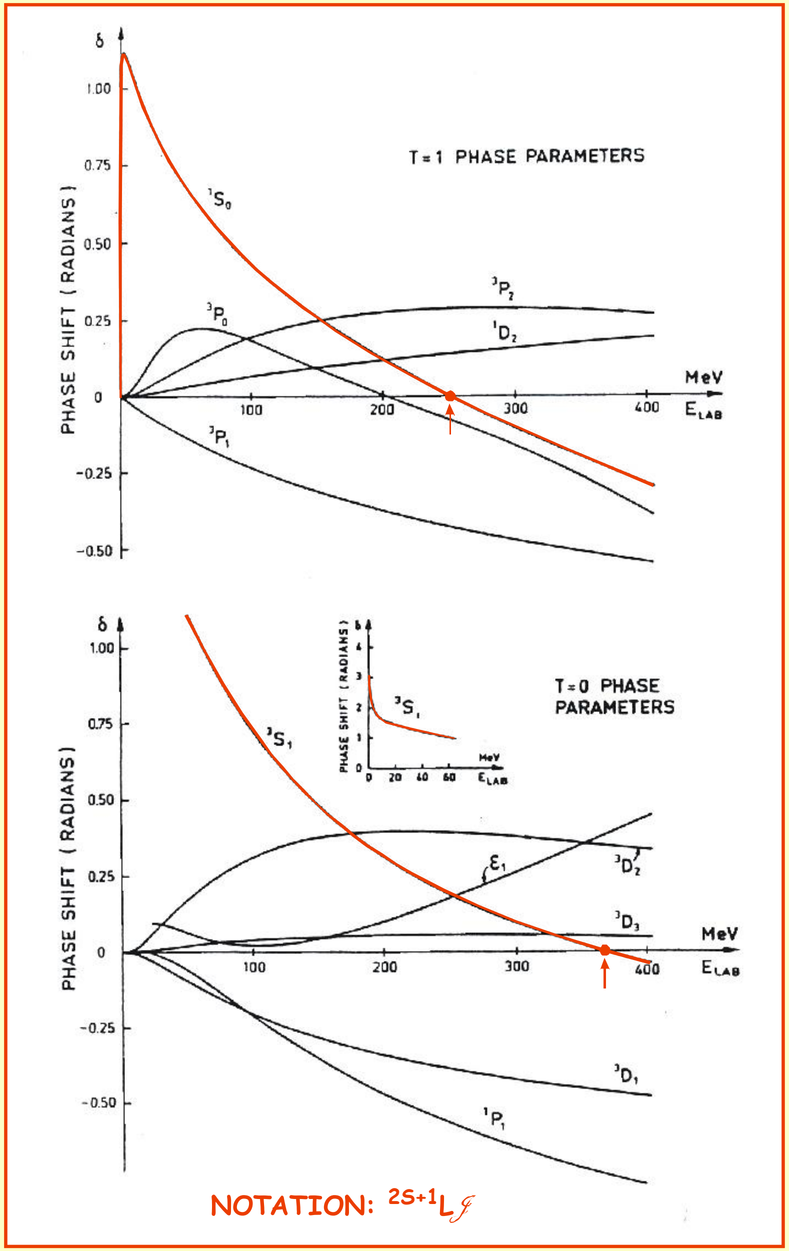
$$S_\ell(k) = e^{2i\delta_\ell(k)}$$

SIMPLE POTENTIAL V(r) SCATTERING: $\delta_\ell(k)$

MORE GENERAL POTENTIALS: $\delta(\ell, S)\, \mathcal{J}(k)$

Phase-shift analysis (with error bars) for a laboratory energy interval 5 MeV $\leq E_{\mathrm{lab}} \leq$ 400 MeV. The phase shifts are given in degrees (taken from (Mac Gregor et al. 68a))

Lab energy (MeV)	$^1S_0$	$^1D_2$	$^1G_4$	$^3P_0$	$^3P_1$	$^3P_2$	ϵ_2
5	54.65 ± 0.03	0.06 ± 0.00	0.00 ± 0.00	1.77 ± 0.02	−1.09 ± 0.01	0.29 ± 0.01	−0.06 ± 0.00
10	54.97 ± 0.07	0.20 ± 0.00	0.00 ± 0.00	3.83 ± 0.04	−2.32 ± 0.01	0.80 ± 0.02	−0.23 ± 0.00
15	53.01 ± 0.09	0.38 ± 0.00	0.01 ± 0.00	5.61 ± 0.07	−3.41 ± 0.02	1.41 ± 0.03	−0.44 ± 0.00
20	50.75 ± 0.11	0.57 ± 0.01	0.03 ± 0.00	7.09 ± 0.10	−4.36 ± 0.03	2.07 ± 0.04	−0.66 ± 0.01
25	48.51 ± 0.11	0.77 ± 0.01	0.05 ± 0.00	8.28 ± 0.12	−5.20 ± 0.04	2.75 ± 0.04	−0.87 ± 0.01
30	46.36 ± 0.11	0.98 ± 0.01	0.07 ± 0.00	9.23 ± 0.14	−5.95 ± 0.05	3.43 ± 0.05	−1.08 ± 0.01
40	42.37 ± 0.12	1.38 ± 0.02	0.12 ± 0.00	10.54 ± 0.18	−7.28 ± 0.05	4.76 ± 0.05	−1.45 ± 0.02
50	38.78 ± 0.13	1.77 ± 0.02	0.17 ± 0.00	11.25 ± 0.20	−8.45 ± 0.06	6.02 ± 0.05	−1.76 ± 0.02
60	35.55 ± 0.14	2.15 ± 0.03	0.23 ± 0.00	11.51 ± 0.22	−9.51 ± 0.06	7.19 ± 0.05	−2.03 ± 0.03
70	32.62 ± 0.16	2.53 ± 0.04	0.29 ± 0.00	11.45 ± 0.23	−10.51 ± 0.07	8.27 ± 0.05	−2.25 ± 0.03
80	29.94 ± 0.18	2.89 ± 0.04	0.35 ± 0.00	11.13 ± 0.23	−11.47 ± 0.07	9.26 ± 0.05	−2.43 ± 0.04
90	27.48 ± 0.20	3.25 ± 0.05	0.41 ± 0.01	10.62 ± 0.23	−12.39 ± 0.07	10.17 ± 0.05	−2.57 ± 0.04
100	25.21 ± 0.21	3.60 ± 0.05	0.47 ± 0.01	9.97 ± 0.23	−13.29 ± 0.07	10.99 ± 0.05	−2.68 ± 0.04
120	21.08 ± 0.23	4.27 ± 0.06	0.59 ± 0.01	8.36 ± 0.23	−15.02 ± 0.07	12.41 ± 0.06	−2.83 ± 0.04
140	17.38 ± 0.25	4.91 ± 0.07	0.71 ± 0.02	6.49 ± 0.24	−16.70 ± 0.09	13.58 ± 0.06	−2.91 ± 0.04
160	13.96 ± 0.27	5.52 ± 0.08	0.82 ± 0.03	4.50 ± 0.26	−18.33 ± 0.11	14.53 ± 0.07	−2.91 ± 0.05
180	10.71 ± 0.29	6.10 ± 0.10	0.93 ± 0.03	2.44 ± 0.30	−19.91 ± 0.13	15.30 ± 0.08	−2.87 ± 0.06
200	7.58 ± 0.31	6.66 ± 0.11	1.04 ± 0.04	0.38 ± 0.34	−21.46 ± 0.16	15.91 ± 0.09	−2.79 ± 0.08
220	4.51 ± 0.34	7.19 ± 0.12	1.15 ± 0.05	−1.65 ± 0.40	−22.96 ± 0.20	16.39 ± 0.10	−2.68 ± 0.09
240	1.46 ± 0.38	7.69 ± 0.14	1.25 ± 0.06	−3.64 ± 0.46	−24.43 ± 0.24	16.77 ± 0.12	−2.55 ± 0.12
260	−1.57 ± 0.43	8.17 ± 0.16	1.35 ± 0.07	−5.57 ± 0.53	−25.86 ± 0.27	17.04 ± 0.15	−2.39 ± 0.14
280	−4.62 ± 0.50	8.63 ± 0.17	1.45 ± 0.08	−7.43 ± 0.60	−27.26 ± 0.30	17.24 ± 0.18	−2.23 ± 0.17
300	−7.67 ± 0.59	9.07 ± 0.19	1.55 ± 0.09	−9.22 ± 0.69	−28.62 ± 0.34	17.37 ± 0.21	−2.05 ± 0.20
320	−10.75 ± 0.71	9.49 ± 0.21	1.65 ± 0.10	−10.93 ± 0.76	−29.94 ± 0.37	17.44 ± 0.24	−1.86 ± 0.23
340	−13.85 ± 0.84	9.89 ± 0.23	1.74 ± 0.11	−12.57 ± 0.83	−31.23 ± 0.40	17.46 ± 0.27	−1.67 ± 0.26
360	−16.97 ± 0.99	10.28 ± 0.25	1.83 ± 0.12	−14.12 ± 0.90	−32.49 ± 0.43	17.44 ± 0.31	−1.47 ± 0.29
380	−20.11 ± 1.15	10.64 ± 0.27	1.92 ± 0.13	−15.61 ± 0.97	−33.72 ± 0.46	17.38 ± 0.35	−1.27 ± 0.32
400	−23.27 ± 1.33	11.00 ± 0.29	2.01 ± 0.14	−17.02 ± 1.04	−34.91 ± 0.49	17.28 ± 0.39	−1.07 ± 0.35



$$\mathrm{tg}\,\delta = -\frac{mk}{\hbar^2}\int_0^\infty V(r)\,j_\ell^2(kr)\,r^2\,dr$$

Born approximation

GENERAL PROPERTIES OF N-N INTERACTION POTENTIALS

a. Hermitian

b. Symmetric for permutation symmetry
$V(1,2) = V(2,1)$

c. Translational invariance $\rightarrow \vec{r} = \vec{r}_1 - \vec{r}_2$

d. Galilean invariance $\rightarrow \vec{p} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$

e. Parity invariant (strong int.)
$V(\vec{r}, \vec{p}, ...) = V(-\vec{r}, -\vec{p}, ...)$

f. Time-reversal invariance
$V(\vec{p}, \vec{\sigma}, ...) = V(-\vec{p}, -\vec{\sigma}, ...)$

g. Rotational invariance
$\Rightarrow r^2, p^2, L^2, \vec{L}\cdot\vec{S}, \quad (\vec{S} = \frac{1}{2}(\vec{\sigma}^{(1)} + \vec{\sigma}^{(2)}))$

h. Rotational invariance (charge-isospin)

- CENTRAL INTERACTION

$$V_C(1,2) = V_C(r) + V_\sigma(r)\,\vec{\sigma}^{(1)}\,.\,\vec{\sigma}^{(2)}$$

$$+ V_\tau(r)\,\vec{\tau}_1\,.\,\vec{\tau}_2 + V_{\sigma\tau}(r)\;\vec{\sigma}_1\,.\,\vec{\sigma}_2\;\;\vec{\tau}_1\,.\,\vec{\tau}_2$$

- TENSOR INTERACTION

$$V_T(1,2) = \left[V_{T_0}(r) + V_{T\tau}(r)\;\vec{\tau}_1\,.\,\vec{\tau}_2\right] S_{1,2}$$

$$\text{with } S_{12} = \frac{3}{r^2}\;(\vec{\sigma}_1\,.\,\vec{r}\,)\;(\vec{\sigma}_2\,.\,\vec{r}\,) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

- SPIN -ORBIT INTERACTION

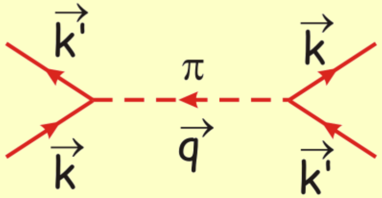
$$V_{LS} = V_{LS}(r)\,\vec{L}\,.\,\vec{S}$$

- QUADRATIC SPIN-ORBIT INTERACTION

ONE-PION EXCHANGE - IMPORTANT PART OF NN INTERACTION

- ELASTIC SCATTERING IN MOMENTUM SPACE

$$V^{\pi NN}(\vec{q} = \vec{k}' - \vec{k}) = \frac{g_\pi^2}{4M^2} \frac{(\vec{\sigma}_i . \vec{q})(\vec{\sigma}_j . \vec{q})}{\vec{q}^2 + m_\pi^2}$$



- POTENTIAL (FOURIER TRANSFORM) IN COORDINATE SPACE

$$V_\pi^{OPEP} = \frac{g_\pi^2}{4M^2} \frac{1}{3} m_\pi \; \vec{\tau}_i . \vec{\tau}_j \left\{ \vec{\sigma}_i . \vec{\sigma}_j + \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right) \underbrace{(3\vec{\sigma}_i . \hat{r} \; \vec{\sigma}_j . \hat{r} - \vec{\sigma}_i . \vec{\sigma}_j)}_{\text{Tensor part}} \right\} \frac{e^{-\mu r}}{\mu r}$$

The Hamada-Johnston Potential

$$V = V_C(r) + V_T(r)S_{12} + V_{LS}(r)\boldsymbol{l}\cdot\boldsymbol{S} + V_{LL}(r)L_{12}\ ,$$

with

$$S_{12} \equiv \frac{3}{r^2}(\boldsymbol{\sigma}_1\cdot\boldsymbol{r})(\boldsymbol{\sigma}_2\cdot\boldsymbol{r}) - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\ ,$$

$$L_{12} \equiv (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\boldsymbol{l}^2 - \tfrac{1}{2}\left[(\boldsymbol{\sigma}_1\cdot\boldsymbol{l})(\boldsymbol{\sigma}_2\cdot\boldsymbol{l}) + (\boldsymbol{\sigma}_2\cdot\boldsymbol{l})(\boldsymbol{\sigma}_1\cdot\boldsymbol{l})\right]\ ,$$

$$\equiv (\delta_{l,J} + \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\boldsymbol{l}^2 - (\boldsymbol{l}\cdot\boldsymbol{S})^2\ .$$

The radial functions are, at large distances, restricted by the condition of approaching the OPEP.

$$V_C(r) = v_0(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)Y(x)\left[1 + a_C Y(x) + b_C Y^2(x)\right]\ ,$$

$$V_T(r) = v_0(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)Z(x)\left[1 + a_T Y(x) + b_T Y^2(x)\right]\ ,$$

$$V_{LS}(r) = g_{LS} v_0 Y^2(x)\left[1 + b_{LS} Y(x)\right]\ ,$$

$$V_{LL}(r) = g_{LL} v_0 \frac{Z(x)}{x^2}\left[1 + a_{LL} Y(x) + b_{LL} Y^2(x)\right]\ ,$$

$$v_0 = \frac{1}{3}\frac{f^2}{\hbar c} m_\pi c^2 = 3.65\ \mathrm{MeV}\ ,$$

$$x = (m_\pi c)/\hbar \cdot r = r/1.43\ \mathrm{fm}\ ,$$

$$Y(x) = \frac{1}{x}\exp(-x)\ ,$$

$$Z(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right)\cdot Y(x)\ .$$

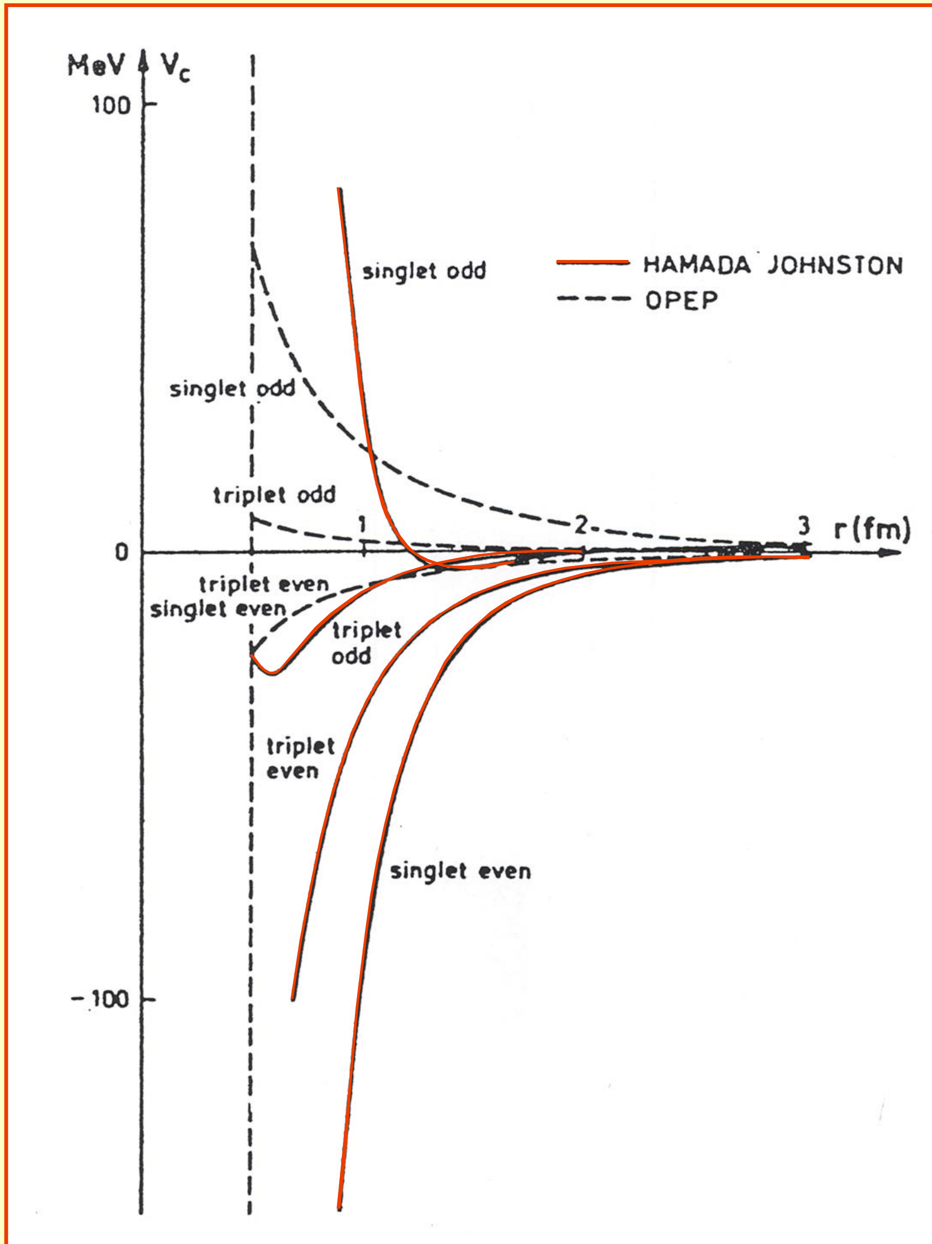
In addition, an infinite repulsion at the radius $c = 0.49$ fm ($x_c = 0.343$), is assumed.

The optimum, adjusted parameters are given in the table.

	Singlet even	Triplet even	Singlet odd	Triplet odd
a_C	8.7	6.0	−8.0	−9.07
b_C	10.6	−1.0	12.0	3.38
a_T	−.	−0.5	−.	−1.29
b_T	−.	0.2	−.	0.55
g_{LS}	−.	2.77	−.	7.36
b_{LS}	−.	−0.1	−.	−7.1
g_{LL}	−0.033	0.1	−0.1	−0.033
a_{LL}	0.2	1.8	2.0	−7.3
b_{LL}	−0.2	−0.4	6.0	6.9

The values of the different potentials at the hard core $r = c$ have been determined as

	V_c	V_T	V_{LS}	V_{LL}
Singlet, even	−1460	−	−	−42
Triplet,even	−207	−642	34	668
Singlet, odd	2371	−	−	−6683
Triplet, odd	−23	173	−1570	−1087



MeV
V_c
100
0
-100
1
2
3
r(fm)
singlet odd
HAMADA JOHNSTON
OPEP
singlet odd
triplet odd
triplet even
singlet even
triplet
odd
triplet
even
singlet even

TWO-NUCLEON (NN) INTERACTIONS

- Argonne V_8, V_{18} potentials

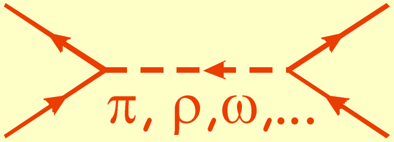
  $\rightarrow$ Coulomb, one-pion exch. (OPEP) + intermediate and short-range

  $$V_{i,j} = \sum_{p=1,18} v_p \, (r_{ij}) \;\; \hat{O}^p_{i,j}$$

  $$\hat{O}^p_{i,j} = \{ 1, \vec{\sigma}_i \cdot \vec{\sigma}_j \, , \vec{S}_{ij} \, , \vec{L} \cdot \vec{S} \, , \ldots \} \otimes \{ 1, \vec{\tau}_i \cdot \vec{\tau}_j \}$$

  Fitted to 4300 nn scattering data

- Bonn potential
  - Based on meson exchange



- Effective field theory (EFT)

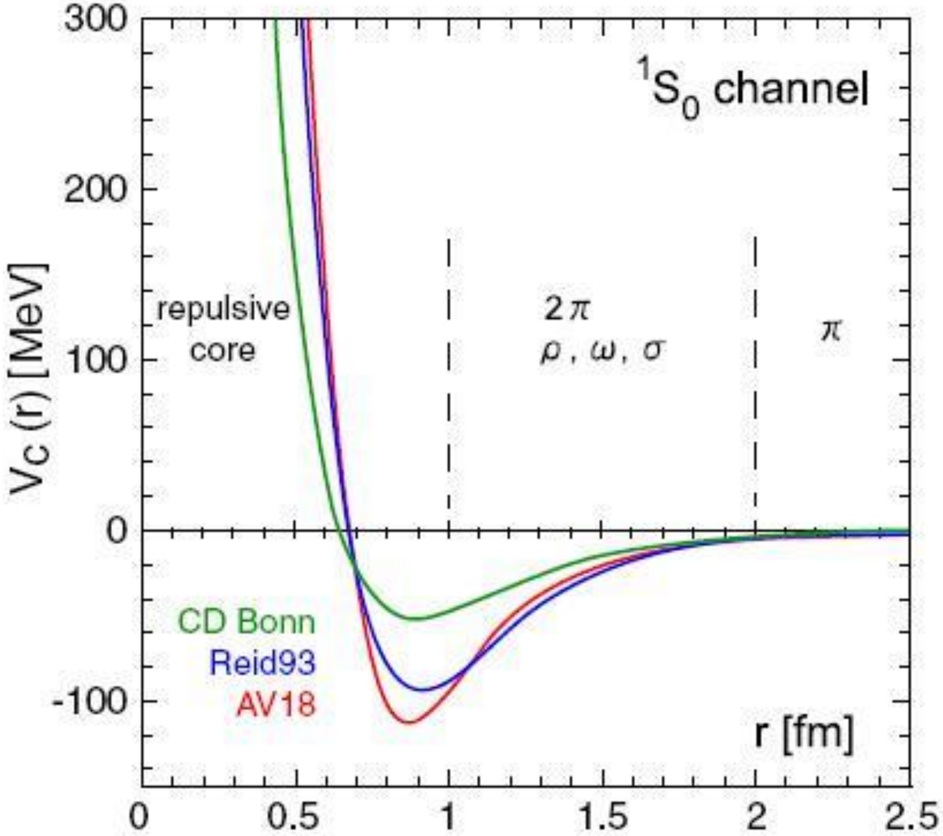
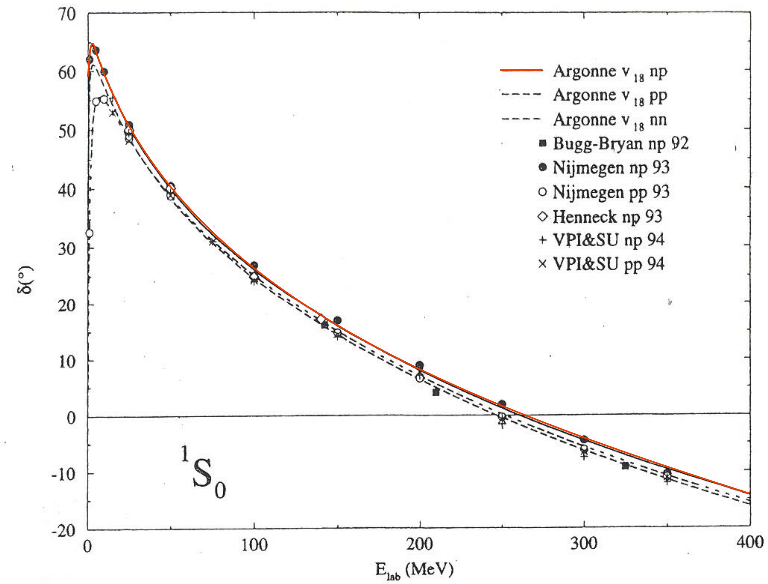
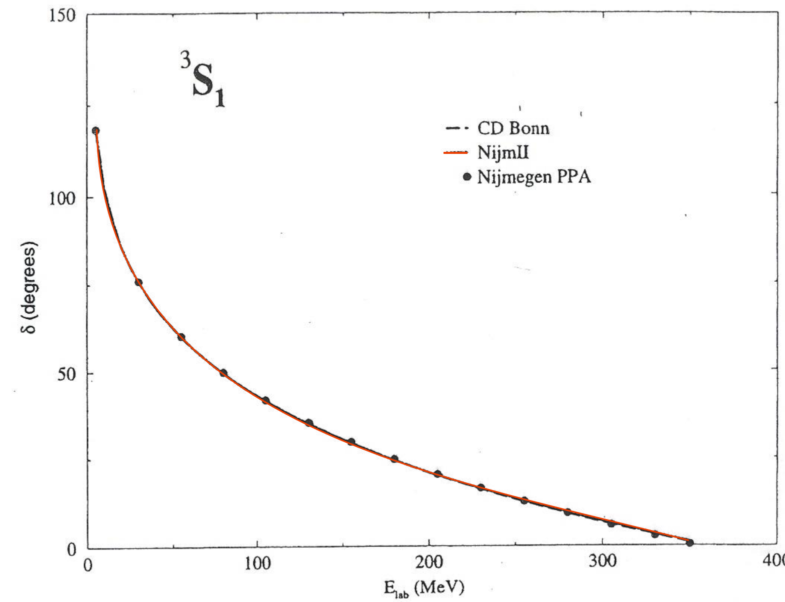


FIG. 1 (color online). Three examples of the modern NN potential in the $^1S_0$ (spin singlet and s-wave) channel: CD-Bonn [17], Reid93 [18], and AV18 [19] from the top at $r = 0.8$ fm.



$^1S_0$ phases of the Argonne v_{18} interaction compared to various *np* and *pp* phase-shift analyses: Argonne v_{18}, Wiringa, Stoks, and Schiavilla, 1995; Bugg-Bryan, Bugg and Bryan, 1992; Nijmegen, Stoks *et al.*, 1993; Henneck, Henneck, 1993; VPI-SU, Arndt, Workman, and Pavan, 1994. Figure from Wiringa *et al.*, 1995.



$^3S_1$ phases from different modern *NN* interaction models: CD Bonn, Machleidt *et al.*, 1996; Nijm II, Stoks, Klomp, *et al.*, 1994; Nijmegan PPA, Stoks, Klomp, *et al.*, 1993. Figure from Wiringa, Stoks, and Schiavilla, 1995.

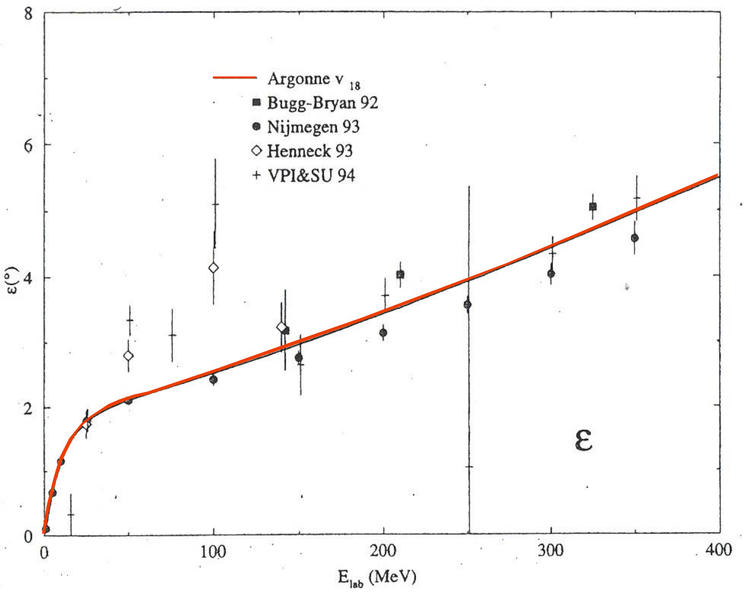


FIG. 4. $^3S_1$-$^3D_1$ mixing parameter ϵ_1 from the Argonne v_{18} interaction and various phase-shift analyses: Argonne v_{18}, Wiringa, Stoks, and Schiavilla, 1995; Bugg-Bryan, Bugg and Bryan, 1992; Nijmegen, Stoks *et al.*, 1993; Henneck, Henneck, 1993; VPI-SU, Arndt, Workman, and Pavan, 1994. Figure from Wiringa *et al.*, 1995.

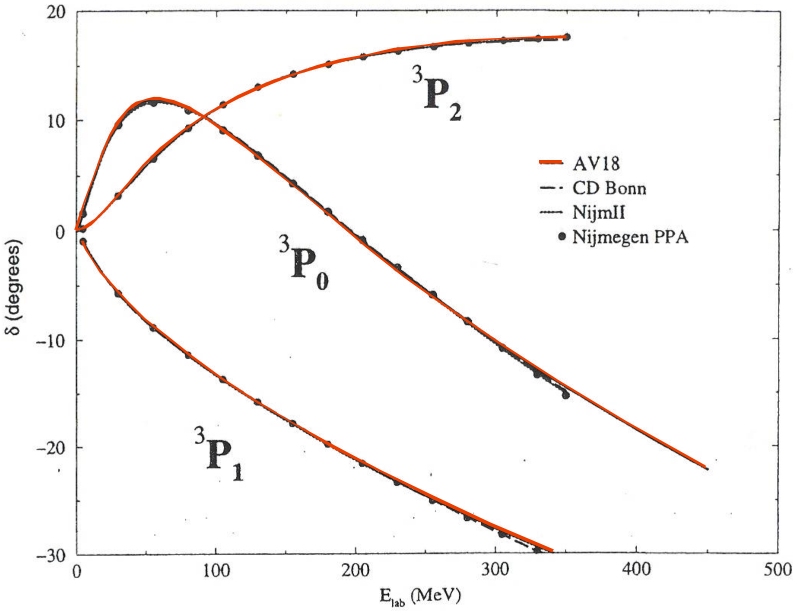
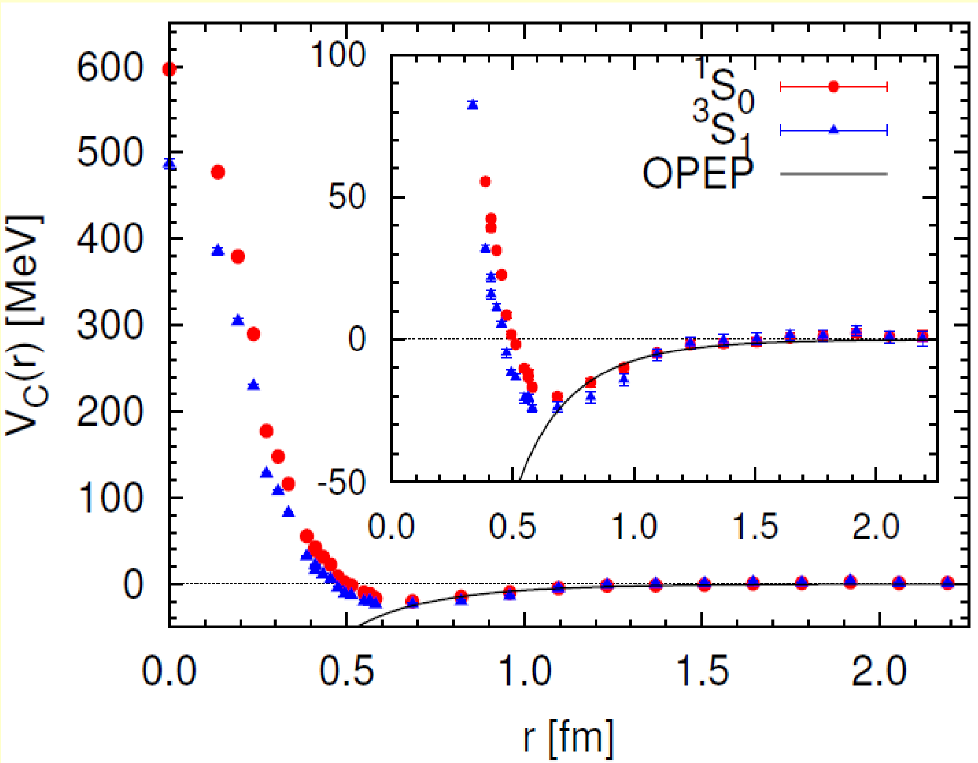


FIG. 6. ^3P_J phases from different modern NN interaction models: AV18, Wiringa *et al.*, 1995; CD Bonn, Machleidt *et al.*, 1996; Nijm II, Stoks, Klomp, *et al.*, 1994; Nijmegan PPA, Stoks, Klomp, *et al.*, 1993. Figure from Wiringa, Stoks, and Schiavilla, 1995.

Nuclear Force from Lattice QCD

N. Ishii[1,2], S. Aoki[3,4] and T. Hatsuda[2]

Phys.Rev.Lett. 99(2007),022001

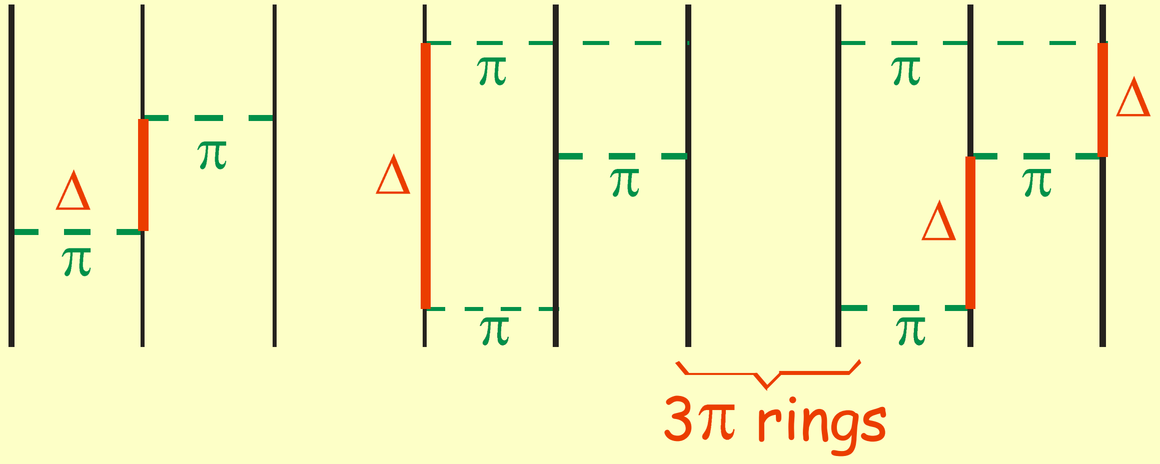


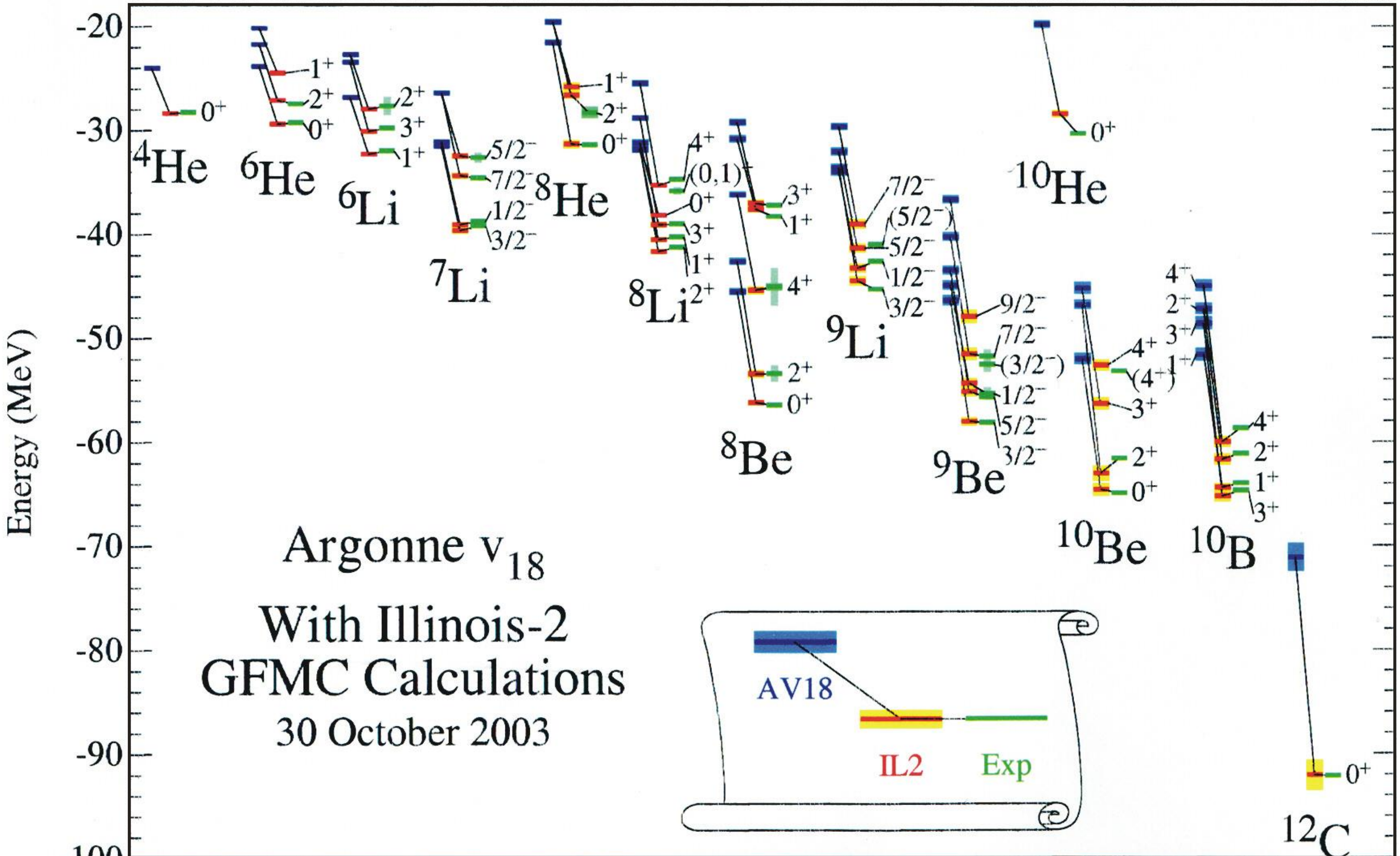
σ.σ τ.τ central force calculated by a Lattice QCD calculation

Calculations for tensor and 3-body forces will be great

THREE-NUCLEON (NNN) INTERACTIONS

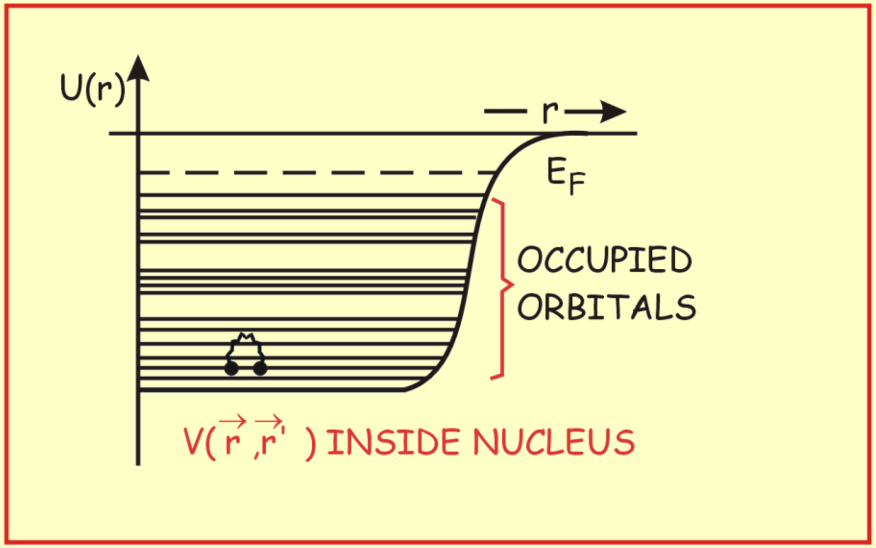
- First evidence for NNN interaction comes from 'exact' calculations for t and ^{3}He.
  - $\rightarrow$ under-'bound' with NN interactions
- Systematic evidence from ab-initio calculations ($A \leq 12$) (Wiringa, Pieper, Pandharipande, Carlsson, Schiavilla)

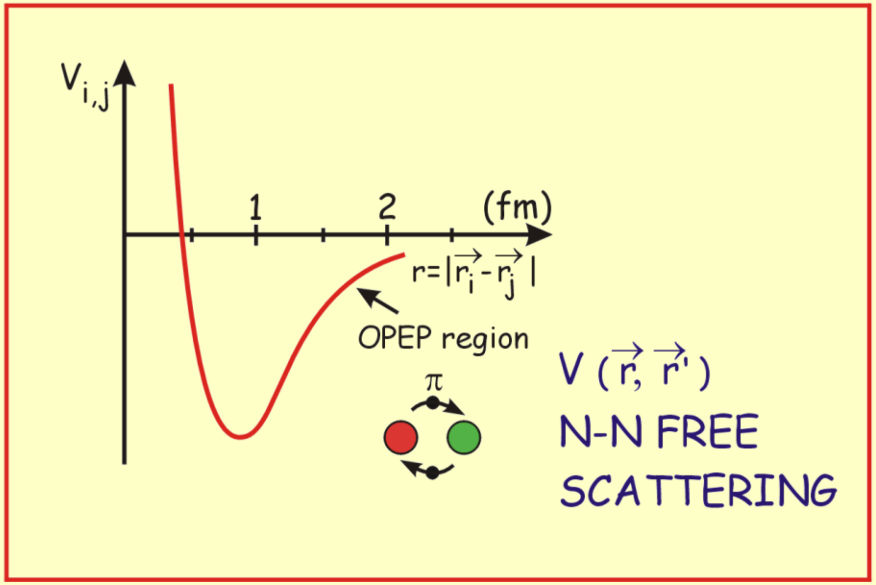




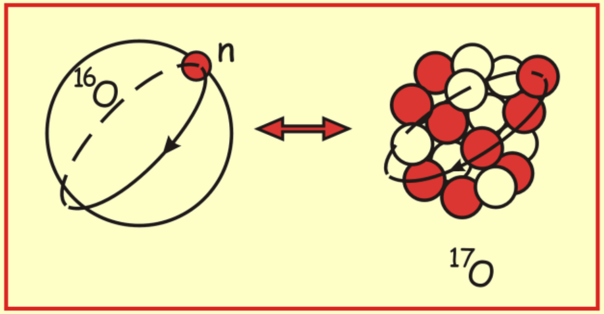
Energy (MeV)
-20
-30
-40
-50
-60
-70
-80
-90
-100
4He
6He
6Li
7Li
8He
8Li
8Be
9Li
9Be
10He
10Be
10B
12C
Argonne v18
With Illinois-2
GFMC Calculations
30 October 2003
AV18
IL2
Exp

Problems using realistic NN forces in nuclei





Concept of effective interaction (operators) active in finite space:



$$\Psi = \sum_i a_i \Psi_i \{17\text{-nucleon coordinates}\}.$$

$$\hat{O} = \sum_{i=1}^{17} \hat{O}\,(\vec{r}_i, \vec{\sigma}_i, \vec{\tau}_i).$$

More general

$(H_0 + V)\Psi = E\Psi \quad \Psi = \sum_{i=1}^{\infty} a_i \Psi_i^{(0)}$

FULL SPACE $(1,\ldots\infty)$

MODEL SPACE $(1,\ldots M)$ $\quad \Psi^M = \sum_{i=1}^{M} a_i \Psi_i^{(0)}$

IMPLICIT EQ. $\langle \Psi^M | H^{eff} | \Psi^M \rangle = E$

FROM REALISTIC (NN free) TOWARDS EFFECTIVE (NN in nucleus) INTERACTION

Some literature (old papers) :

- B. H. Brandow, Rev. Mod. Phys. 39 (1967),771
- B. L. Scott and S. A. Moszkowski, Ann. Phys. 11 (1960), 65
- H. A. Bethe, B. H. Brandow, A. G. Petscheck, PCR 129 (1963), 225
- T.T.S. Kuo and G. E. Brown, Nucl. Phys. 85 (1966), 40; ibid. A103 (1967), 71
- B. R. Barrett and M. W. Kirson, Adv. Nucl. Phys. 6 (1974) 219
  (many more)

Recent:

- D. J. Dean, et al., Progr. Part. Nucl. Phys. 53 (2004), 419

Effective operators

Perturbation theory

Hilbert space → Valence space

$$V_{NN} \Rightarrow V^{eff}$$

$$\hat{H}\,|\Psi\rangle = E|\Psi\rangle \Rightarrow \hat{H}^{eff}\,|\Psi_{eff}\rangle = E|\Psi_{eff}\rangle$$

$$\langle\Psi|\hat{O}|\Psi\rangle \Rightarrow \langle\Psi_{eff}|\hat{O}_{eff}|\Psi_{eff}\rangle$$

- *microscopic* effective interaction (*realistic* interaction)

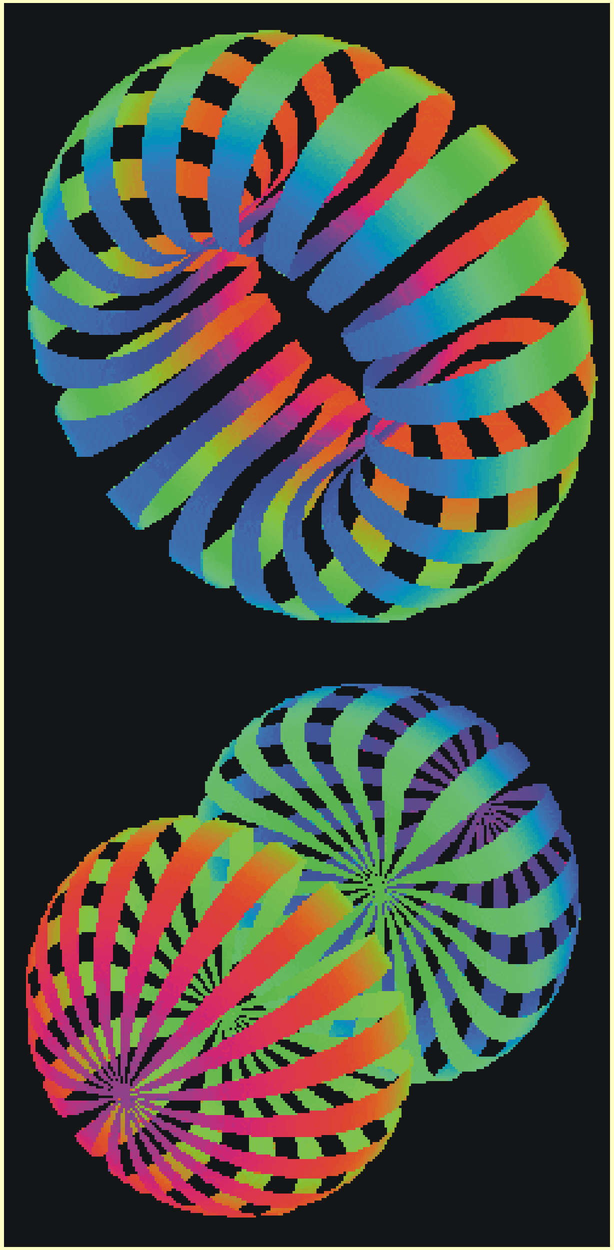
$$V_{NN} \Rightarrow \mathrm{G} \Rightarrow V^{eff}$$

M. Hjorth-Jensen et al, Phys.Rep.261 (1995)

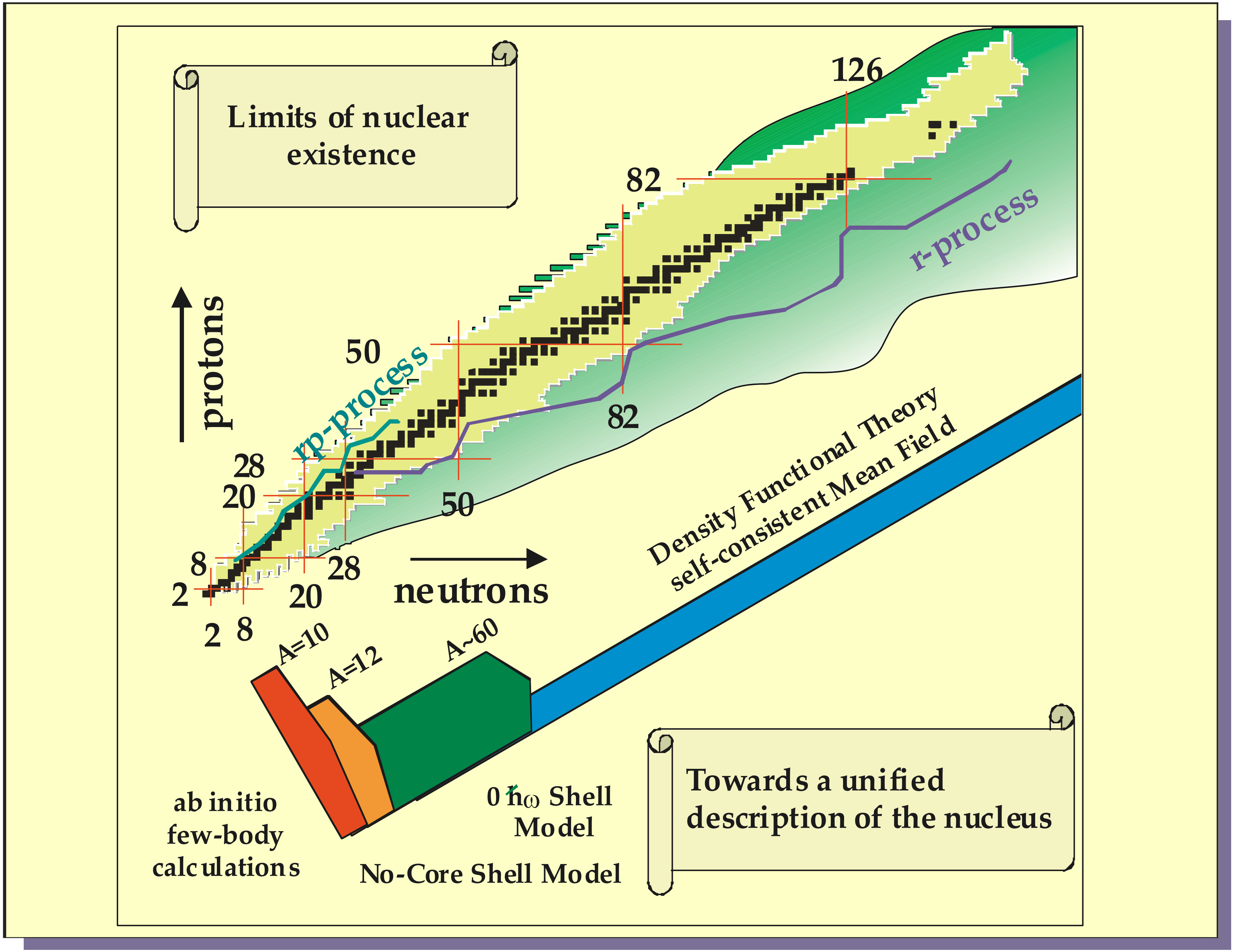
- *empirical* interaction (fitted to the data)

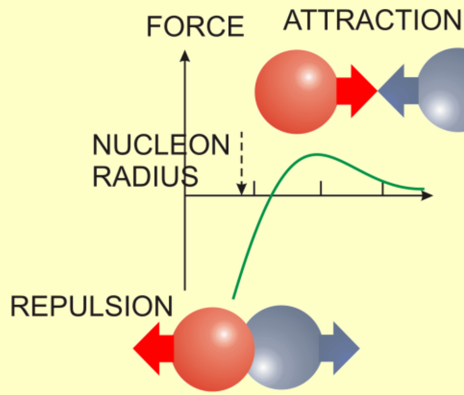
B.A.Brown, B.H.Wildenthal, Ann. Rev. Nucl.Part.Sci. 38 (1988)

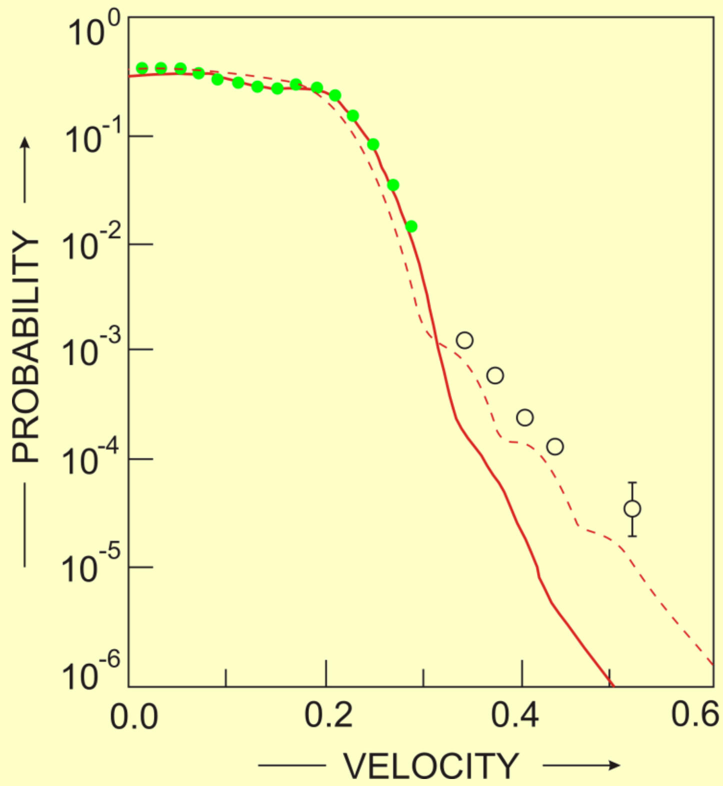
- *schematic* interaction (delta-force, etc)



Carlsson, Schiavilla,
Rev. Mod. Phys. 70(1998), 743


Limits of nuclear existence
126
82
50
28
20
8
2
rp-process
r-process
protons
neutrons
82
50
28
20
2
8
Density Functional Theory
self-consistent Mean Field
A=10
A=12
A~60
ab initio
few-body
calculations
0 ħω Shell
Model
No-Core Shell Model
Towards a unified
description of the nucleus





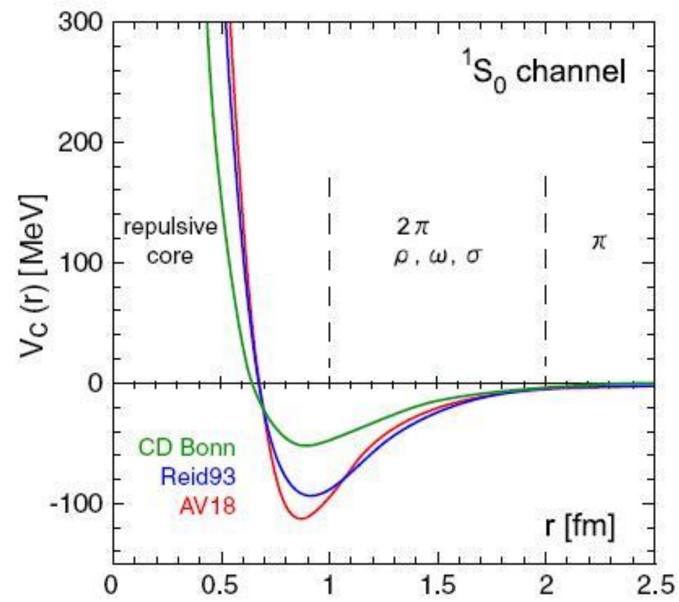


FIG. 1 (color online). Three examples of the modern *NN* potential in the $^1S_0$ (spin singlet and *s*-wave) channel: CD-Bonn [17], Reid93 [18], and AV18 [19] from the top at $r =$ 0.8 fm.