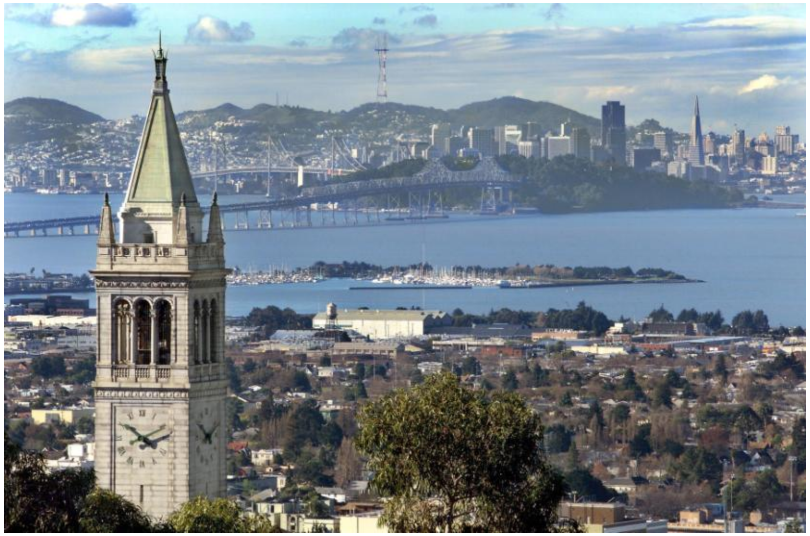
Where I am coming from …

My hometown : Wuppertal

My first university : Bochum

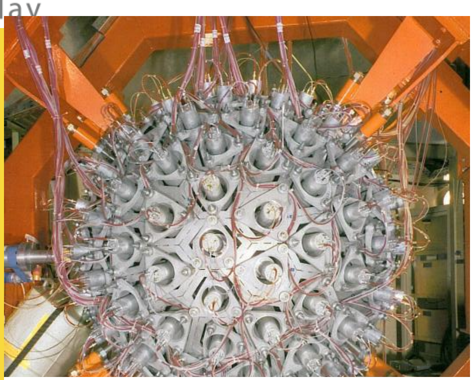
My further studies : Heidelberg

My postdoctoral years : Berkeley



My physics interest: nuclear spectroscopy

Crystal Ball
1983-1989

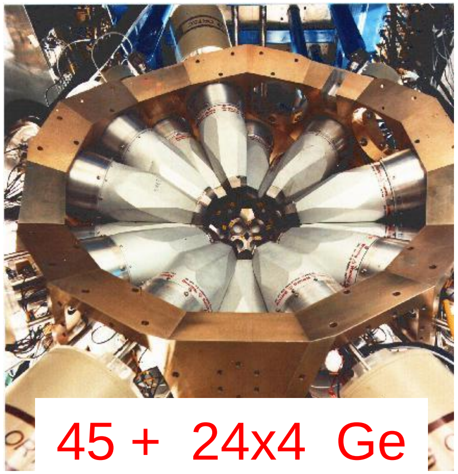


162 NaI detectors

HERA/NORDBALL
1989-1992

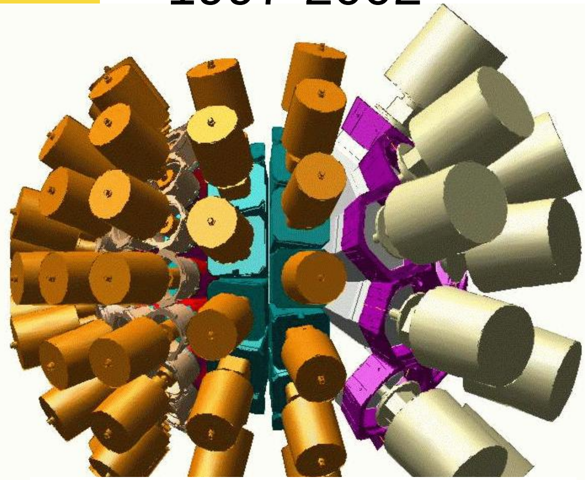
20 Ge detectors

EUROGAM
1992-1996



45 + 24x4 Ge

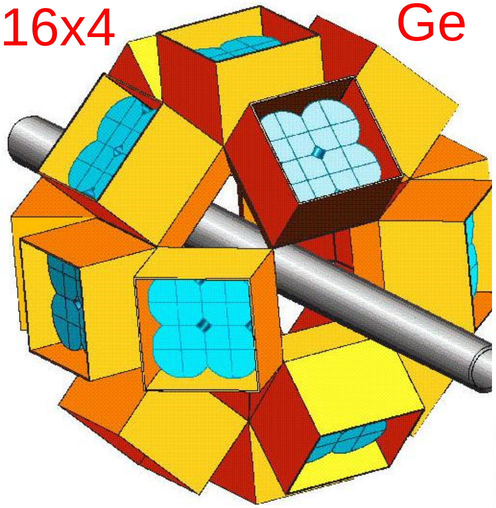
EUROBALL
1997-2002



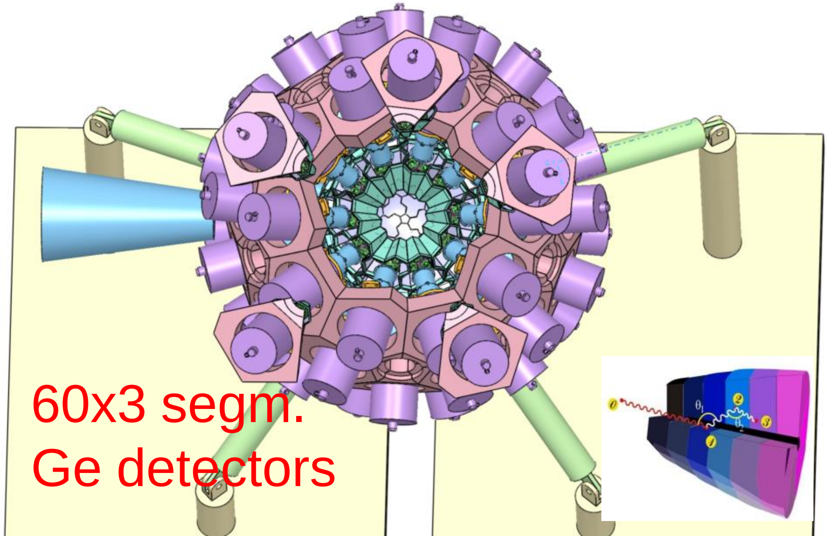
45 + 26x4+ 15x7 Ge

EXOGAM
since 2002

16x4 Ge

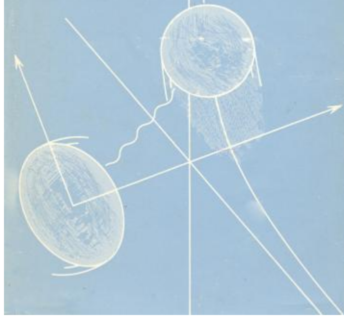


AGATA
demonstrator since 2009



60x3 segm.
Ge detectors

Coulomb excitation - a tool for nuclear shapes and more



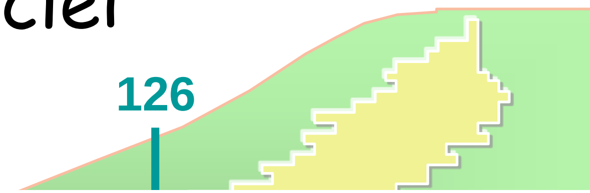
- Introduction
- Theoretical aspects of Coulomb excitation
- Experimental considerations, set-ups and analysis techniques
- Recent highlights and future perspectives

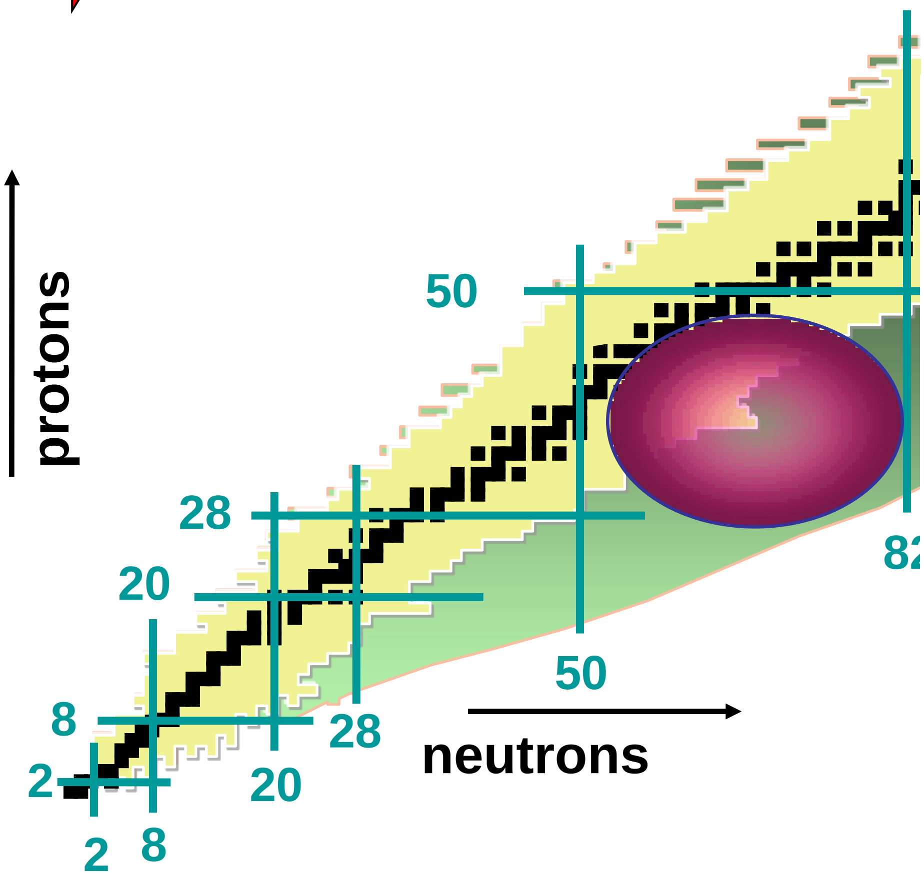
Lecture given at the
Ecole Joliot Curie 2012
Wolfram KORTEN (w.korten@cea.fr)
CEA Saclay

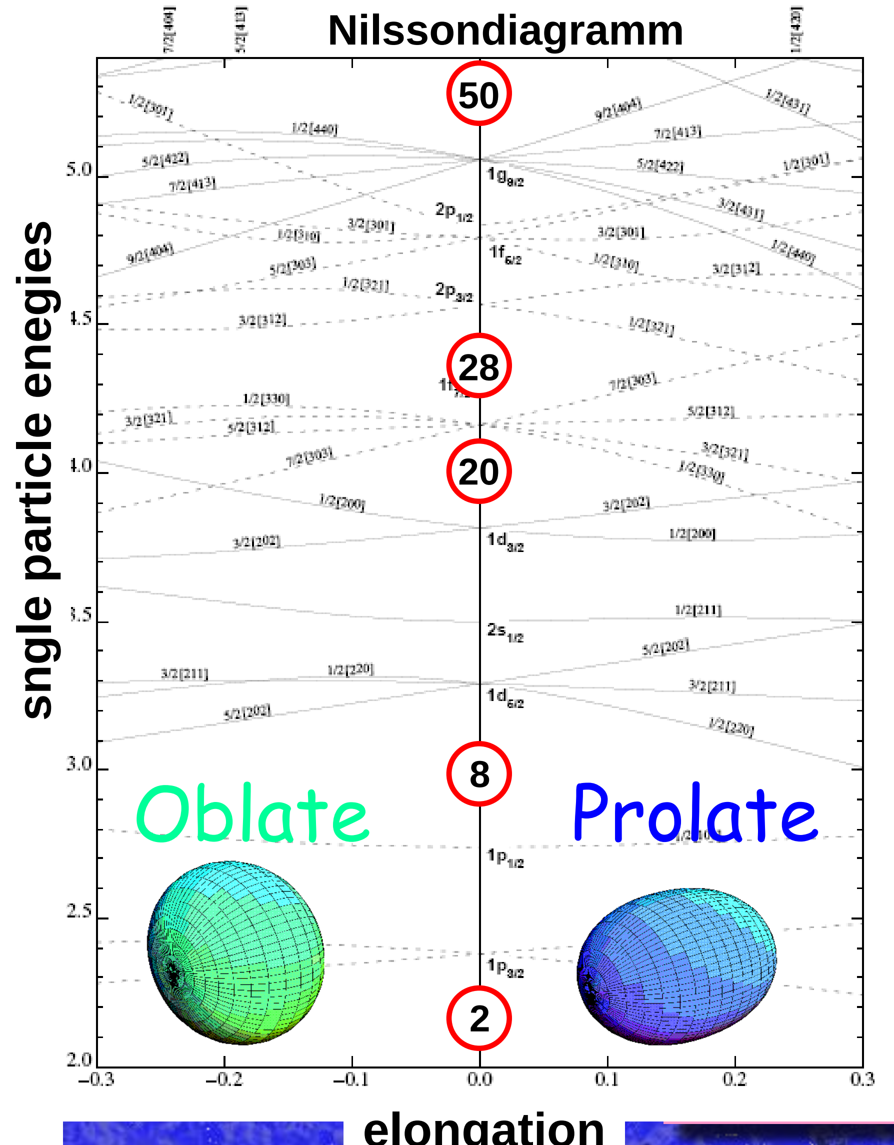
Shapes of atomic nuclei

Z, N = magic numbers

Closed shell = spherical shape

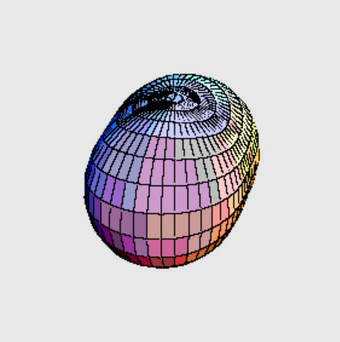


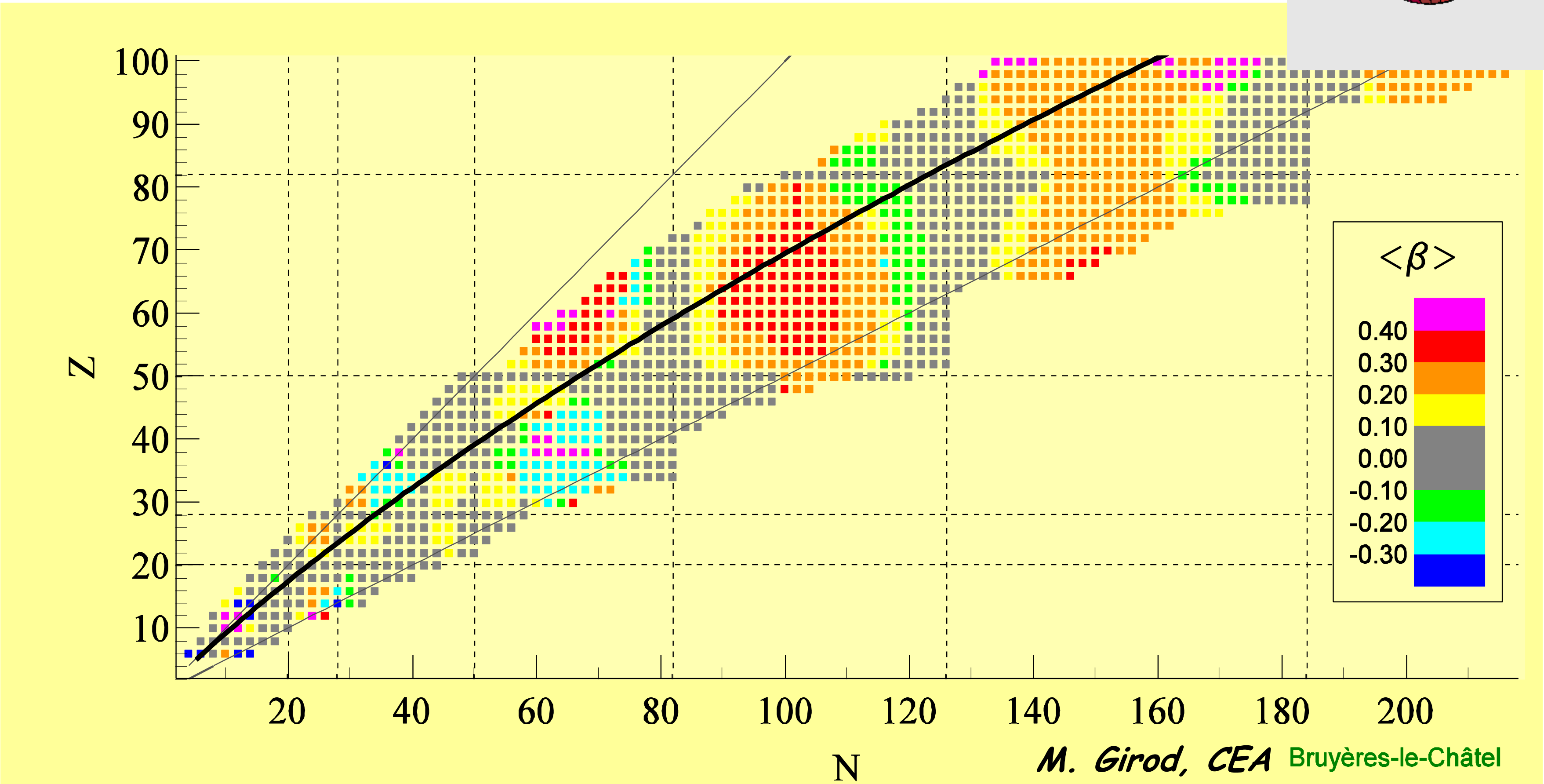




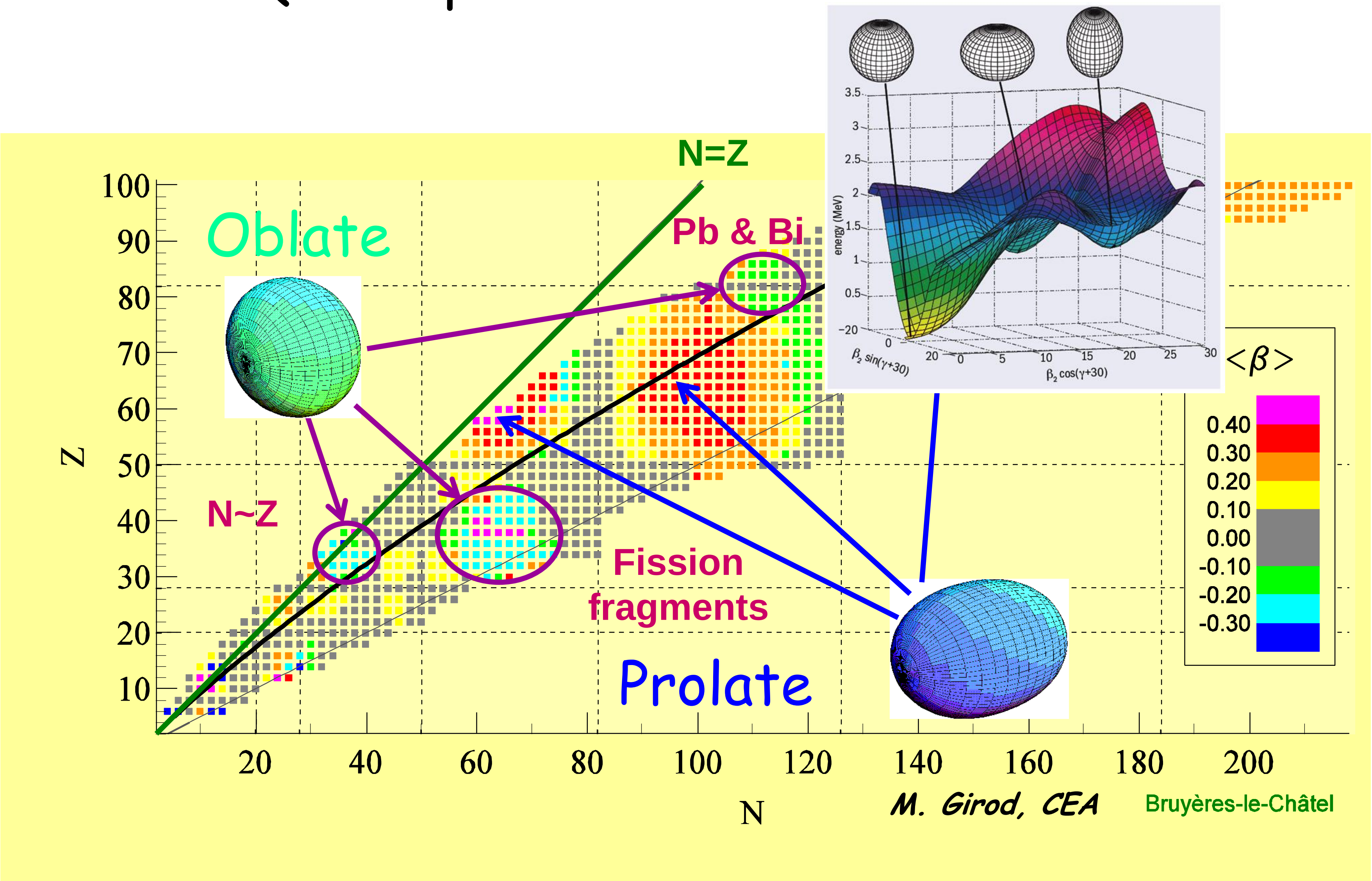
The vast majority of all nuclei shows a non-spherical mass distribution

Quadrupole deformation of nuclei



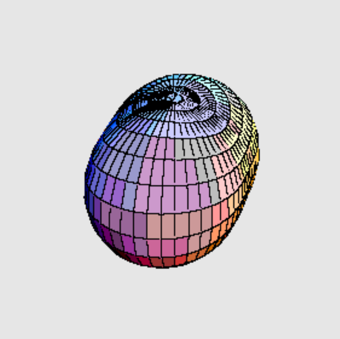


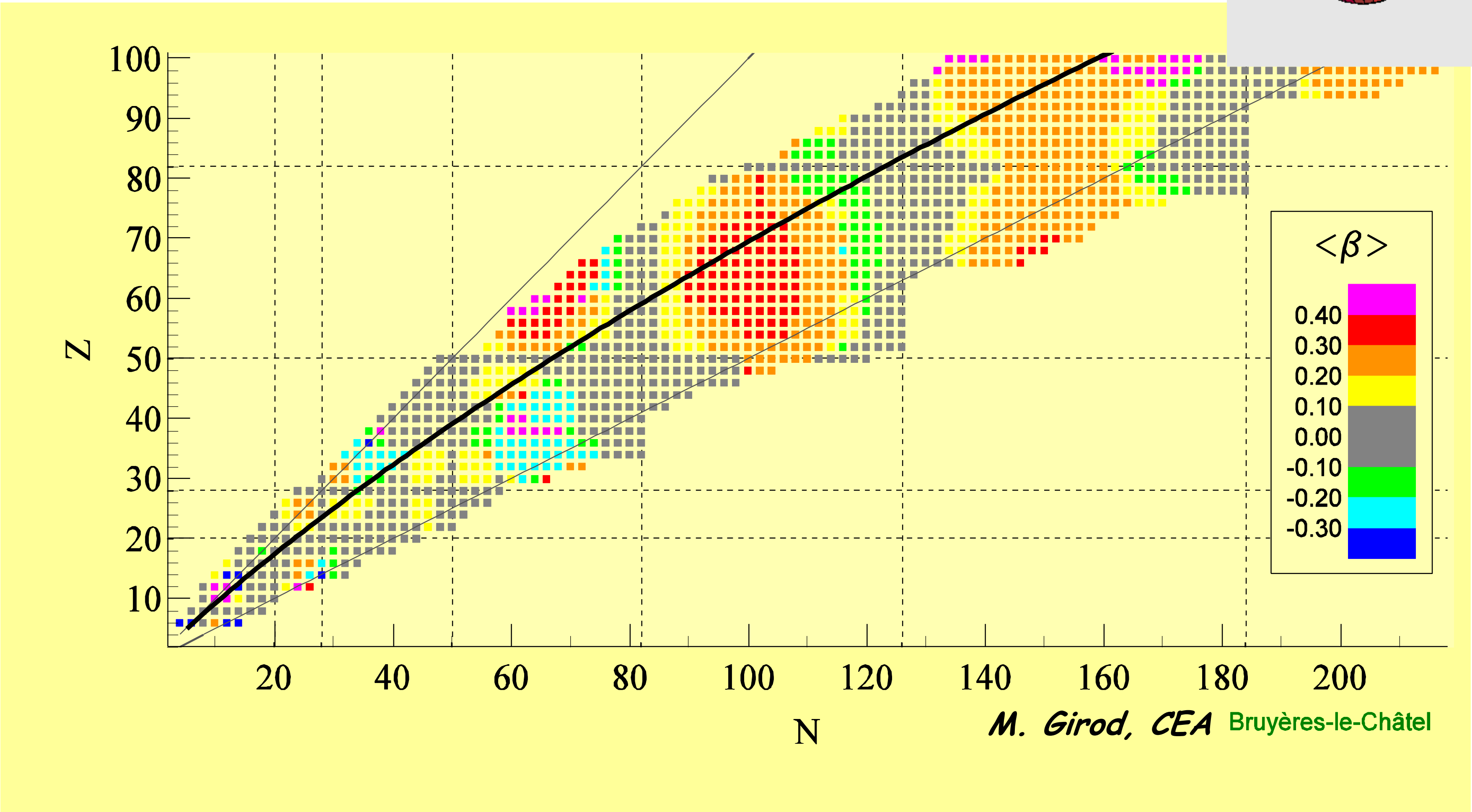
Quadrupole deformation of nuclei



Oblate deformed nuclei are far less abundant than **prolate nuclei**
Shape coexistence possible for certain regions of N & Z

Quadrupole deformation of nuclei





Coulomb excitation excites “collective” degrees of freedom (rotation, vibration) and, in principal, can map the shape of all atomic nuclei (ground and excited states)

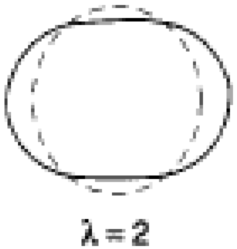
Nuclear shapes and "deformation" parameters

Generic nuclear shapes can be described by a development of spherical harmonics

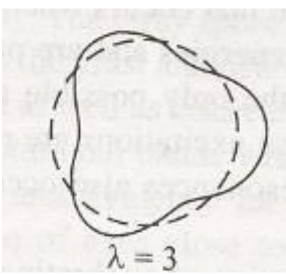
$$R(t) = R_0 \left[1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} a_{\lambda\mu}(t)\, Y_{\lambda\mu}(\vartheta, \varphi) \right]$$

quadrupole $a_{20} = \beta_2 \cos\gamma \qquad a_{22} = a_{2-2} = \frac{1}{\sqrt{2}} \beta_2 \sin\gamma$

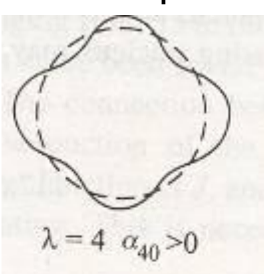
$\alpha_{\lambda\mu}$:deformation parameters



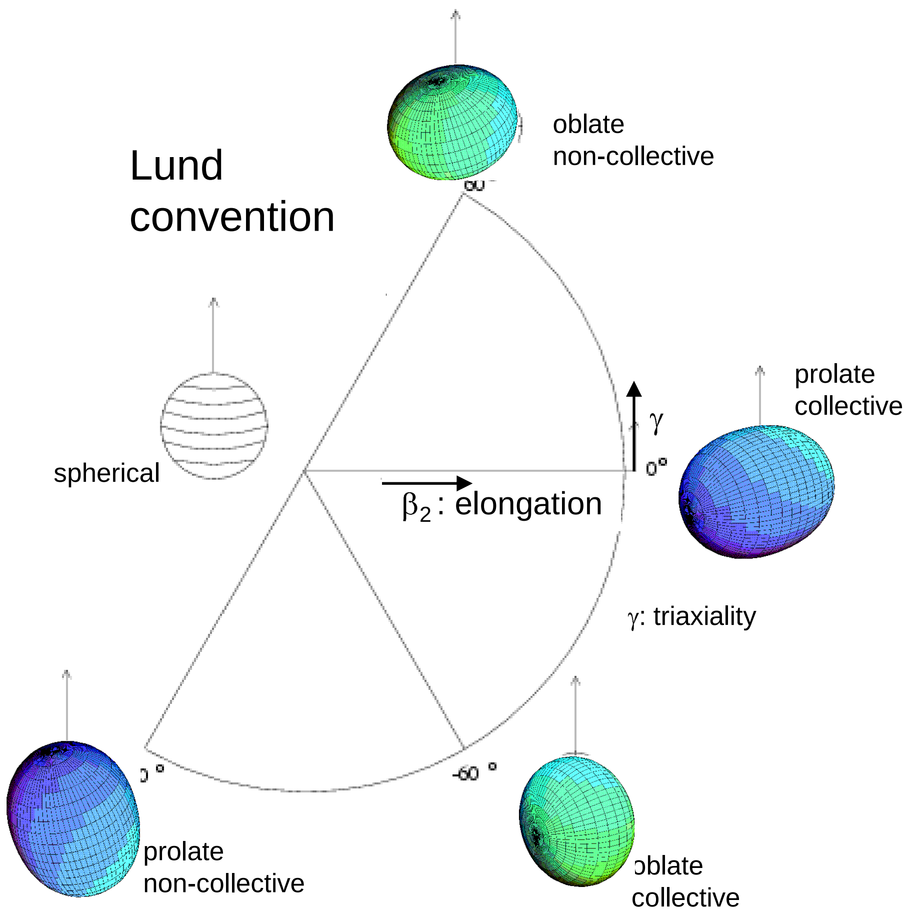
octupole



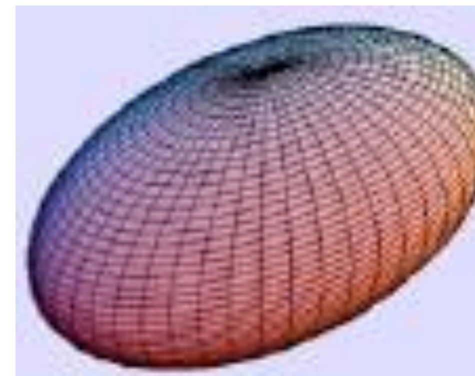
hexadecapole

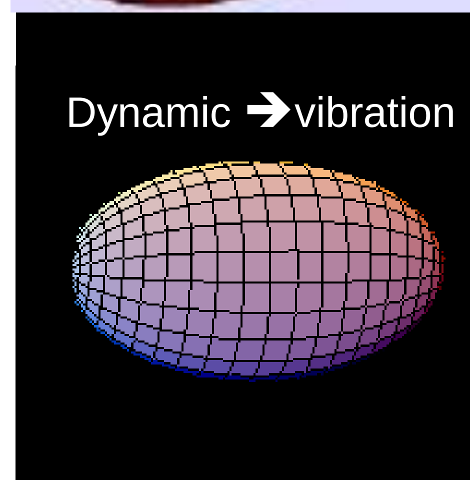


Lund convention



Triaxial Y_{22} deformation / Tetrahedral Y_{32} deformation





Nuclear shapes and electric multipole moments

Electric multipole moments can be expanded in terms of spherical harmonics

$$M(E\lambda,\mu) \equiv Q_{\lambda,\mu} = \sqrt{\frac{2\lambda+1}{16\pi}} \int_0^R \rho(r) r^{\lambda} Y_{\lambda\mu}(\theta,\varphi)\, r^2 dr d\Omega$$

Using the deformation parameters ($\alpha_{\lambda\mu}$) for the the nuclear mass distribution

$$R(t) = R_0 \left[1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} a_{\lambda\mu}(t)\, Y_{\lambda\mu}(\vartheta,\varphi) \right]$$

For axially symmetric shapes ($\beta_\lambda = \alpha_{\lambda 0}$) and a homogenous density distribution ρ

the quadrupole, octupole and hexadecupole moments (Q_2,Q_3,Q_4) become:

$$Q_2 = \sqrt{\frac{3}{5\pi}} Z R_0^2 \left(\beta_2 + 0.360\beta_2^2 + 0.336\beta_3^2 + 0.328\beta_4^2 + 0.967\beta_2\beta_4 + O(\beta^3) \right) \left[fm^2 \right]$$

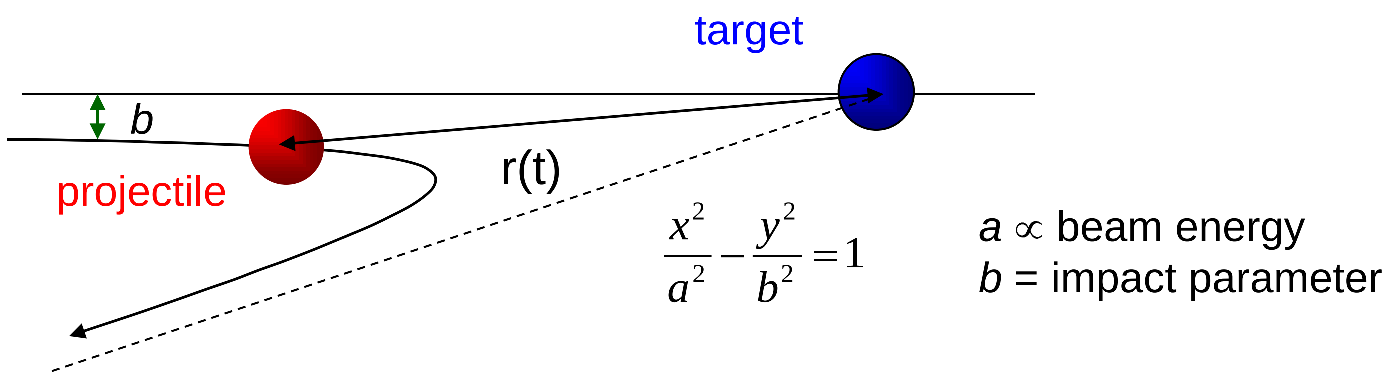
$$Q_3 = \sqrt{\frac{3}{7\pi}} Z R_0^3 \left(\beta_3 + 0.841\beta_2\beta_3 + 0.769\beta_3\beta_4 + O(\beta^3) \right) \left[fm^3 \right]$$

$$Q_4 = \sqrt{\frac{1}{\pi}} Z R_0^4 \left(\beta_4 + 0.725\beta_2^2 + 0.462\beta_3^2 + 0.411\beta_4^2 + 0.983\beta_2\beta_4 + O(\beta^3) \right) \left[fm^4 \right]$$

$$Q_1 = C_{LD} Z A \left(\beta_2\beta_3 + 1.46\beta_3\beta_4 + O(\beta^3) \right) \left[fm \right]$$

Coulomb excitation – an introduction

Rutherford scattering – some reminders

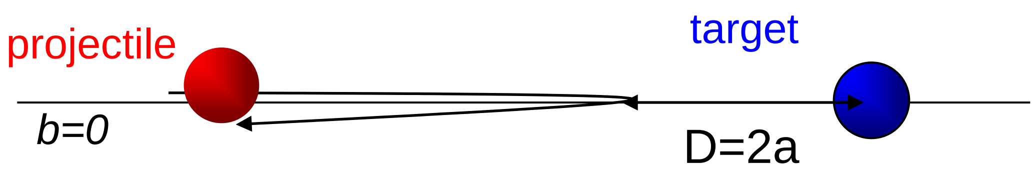


- Elastic scattering of charged particles (point-like ➔ monopoles) under the influence of the Coulomb field
  $F_C = Z_1 Z_2 e^2/r^2$ with $r(t) = |\mathbf{r_1}(t) - \mathbf{r_2}(t)|$
  ➔ hyperbolic relative motion of the reaction partners

- Rutherford cross section
  $d\sigma/d\theta = Z_1 Z_2 e^2/E_{cm}^2 \sin^{-4}(\theta_{cm}/2)$

$$\text{valid as long as } E_{cm} = m_0 v^2 = \frac{m_P \cdot m_T}{m_P + m_T} v^2 << V_c = Z_1 Z_2 e^2/R_{int}$$

Validity of classical Coulomb trajectories

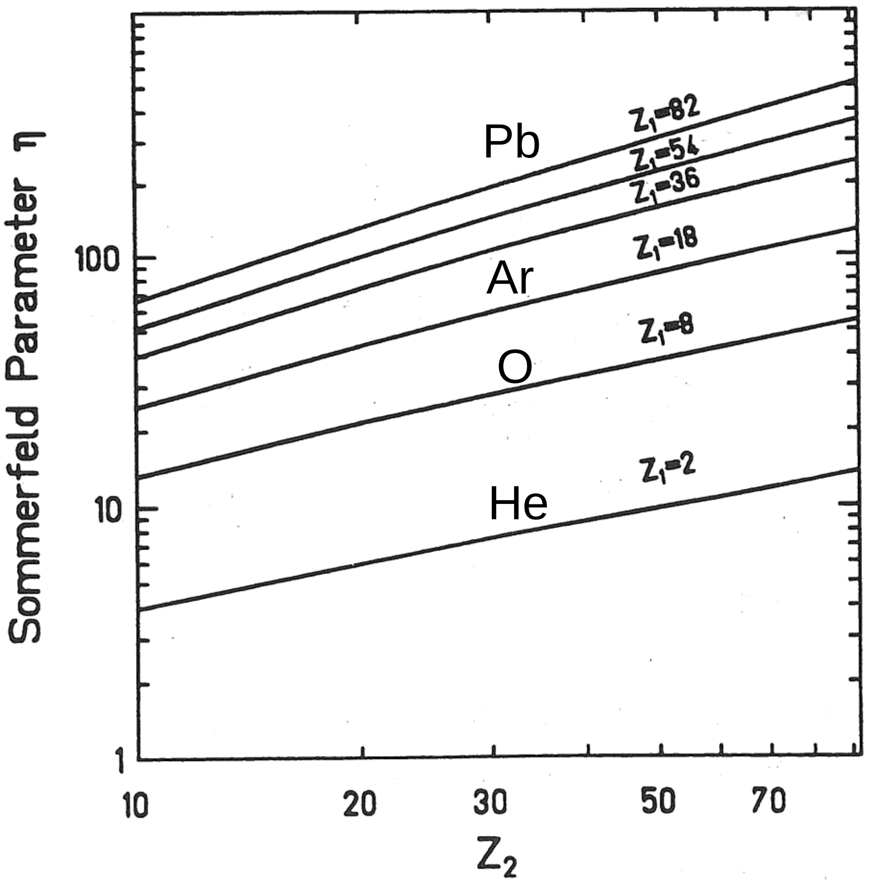


Sommerfeld parameter

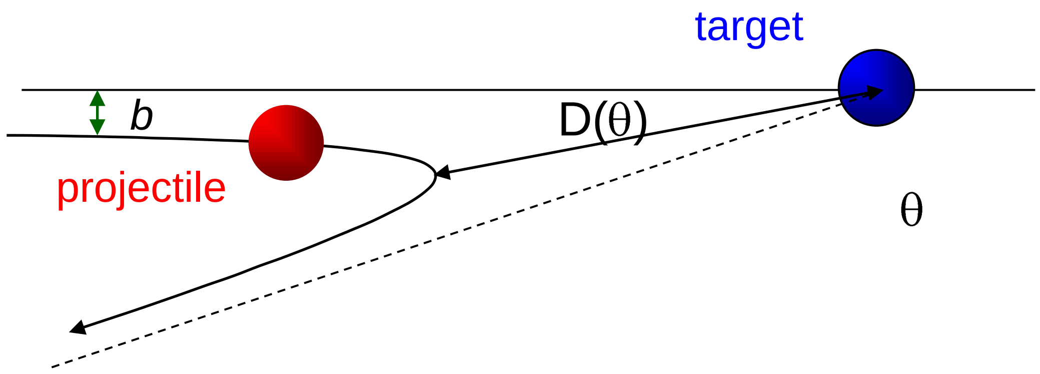
$$\eta = \frac{a}{\lambda\!\!\!^{-}} = \frac{Z_P Z_T e^2}{\hbar v_\infty} >> 1$$

η >> 1 requirement for a semi classical treatment of equations of motion

- measures the strength of the monopole-monopole interaction
- equivalent to the number of exchanged photons needed to force the nuclei on a hyperbolic orbit



Coulomb trajectories - some more definitions



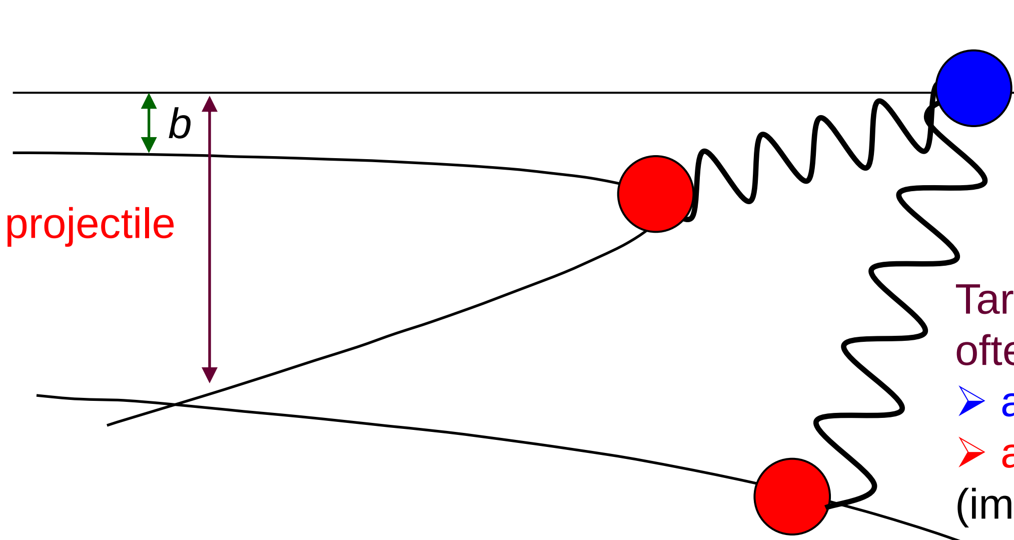
r (w) = a (ε sinh w + 1)
t (w) = a/v_∞ (ε cosh w + w)
a = $Z_p Z_t e^2 E^{-1}$

Principal assumption η>>1 ➔ classical description of the relative motion of the center-of-mass of the two nuclei → hyperbolic trajectories

- distance of closest approach (for w=0): $$D(\theta_{cm}) = a(1+\varepsilon) = a\left[1+\sin\left(\frac{\theta_{cm}}{2}\right)^{-1}\right]$$
- impact parameter: $$b = \sqrt{D^2 - 2aD} = a \cdot \cot\left(\frac{\theta_{cm}}{2}\right)$$
- angular momentum : $$L = \hbar\eta\sqrt{\varepsilon^2 - 1} = \hbar\eta\cot\left(\frac{\theta_{cm}}{2}\right)$$

Coulomb excitation – some basics

Nuclear excitation by the electromagnetic interaction acting between two colliding nuclei.



Target and projectile excitation possible often heavy, magic nucleus (e.g. ^{208}Pb)

- as projectile ➔ target Coulomb excitation
- as target ➔ projectile Coulomb excitation

(important technique for radioactive beams)

small impact parameter

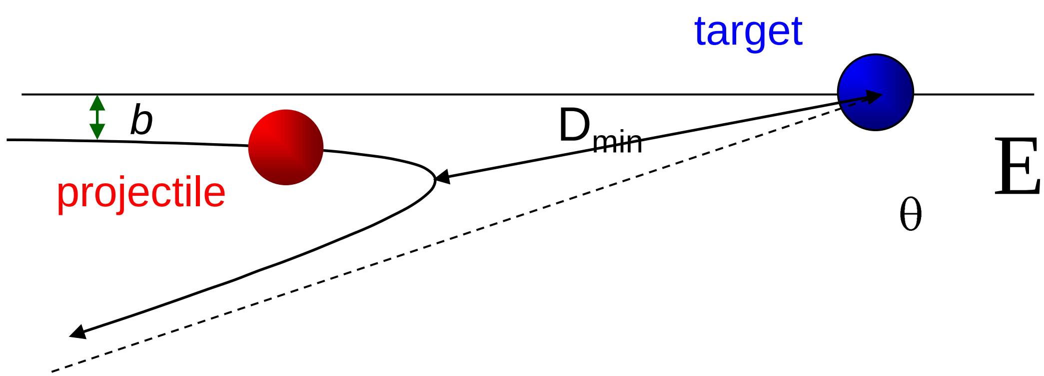
- back scattering
- close approach
- strong EM field

large impact parameter

- forward scattering
- large distance
- weak EM field

Coulomb trajectories only if the colliding nuclei do not reach the "Coulomb barrier" ➔ purely electromagnetic process, no nuclear interaction, calculable with high precision

„Safe“ energy requirement



$$E_{cm}[MeV] = \frac{Z_P Z_T e^2}{D_{min}[fm]}$$

- Rutherford scattering only if the distance of closest approach is large compared to nuclear radii + surfaces:

  „Classical“ approach using the liquid-drop model

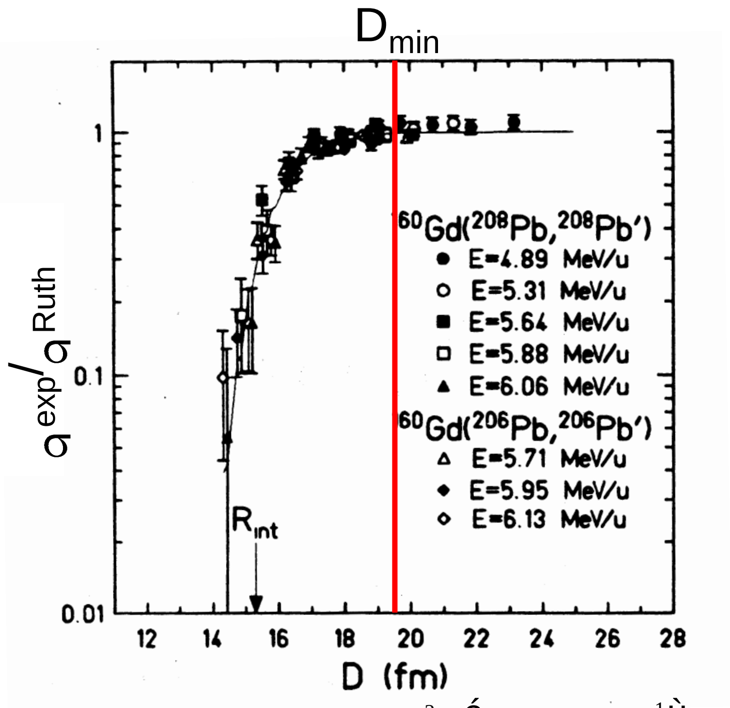
  $$D_{min} \geq r_s = [1.25\,(A_1^{1/3} + A_2^{1/3}) + 5]\ \text{fm}$$

- More realistic approximation using the half-density radius of a Fermi mass distribution of the nucleus :

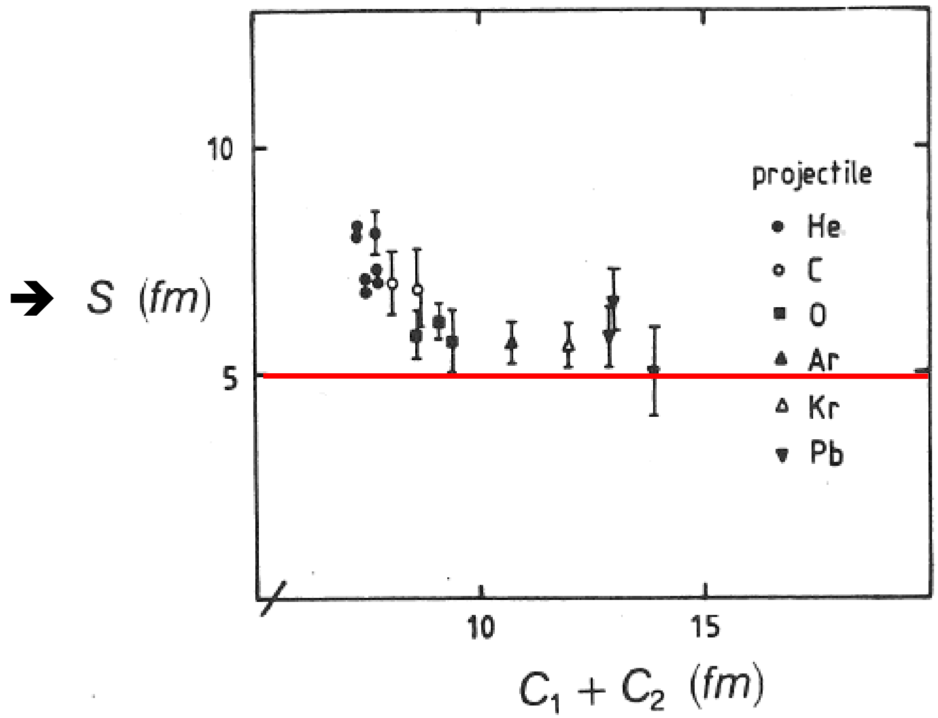
  $C_i = R_i(1-R_i^{-2})$ with $R = 1.28\,A^{1/3} - 0.76 + 0.8\,A^{-1/3}$

  $$\Rightarrow D_{min} \geq r_s = [\,C_1 + C_2 + S\,]\ \text{fm}$$

„Safe“ energy requirement



$$D(\theta_{cm}) = \frac{Z_1 Z_2 e^2}{E_{cm}} \left[1+\sin\left(\frac{\theta_{cm}}{2}\right)^{-1}\right]$$

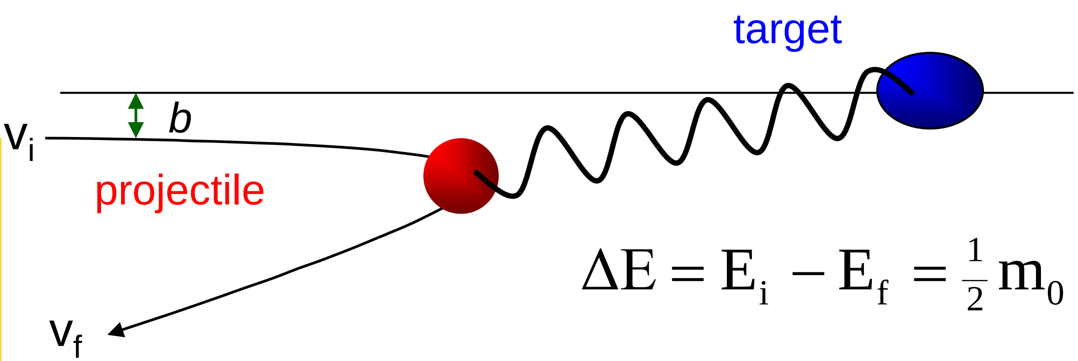


Empirical data on surface distance S as function of half-density radii C_i require distance of closest approach S > 5 - 8 fm

➔ choose adequate beam energy ($D > D_{min}$ for all θ)
low-energy Coulomb excitation

➔ limit scattering angle, i.e. select impact parameter $b > D_{min}$,
high-energy Coulomb excitation

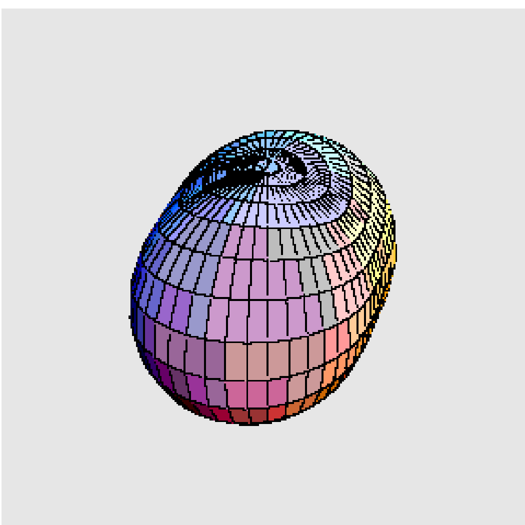
Coulomb excitation – the principal process



$$\Delta E = E_i - E_f = \tfrac{1}{2} m_0 \left(v_i^2 - v_f^2\right) \text{ with } m_0 = \frac{m_P \cdot m_T}{m_P + m_T}$$

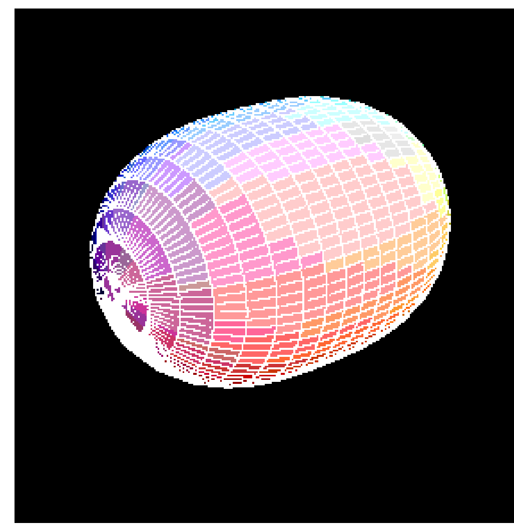
Inelastic scattering: kinetic energy is transformed into nuclear excitation energy

e.g. rotation vibration



Excitation probability (or cross section) is a measure of the collectivity of the nuclear state of interest

➔ complementary to, e.g., transfer reactions



Coulomb excitation – "sudden impact"

Excitation occurs only if nuclear time scale is long compared to the collision time:
„sudden impact" if $\tau_{nucl} >> \tau_{coll} \sim a/v \approx 10$ fm / 0.1c $\approx 2\text{-}3{\cdot}10^{-22}$ s
$\tau_{coll} \sim \tau_{nucl} \sim \hbar/\Delta E$ ➔ adiabatic limit for (single-step) excitations

$$\xi = \frac{\Delta E}{\hbar} \cdot \tau_{coll} = \frac{\Delta E}{\hbar} \frac{a}{v_\infty} = \frac{Z_1 Z_2 e^2}{\hbar} \left(\frac{1}{v_f} - \frac{1}{v_i} \right)$$

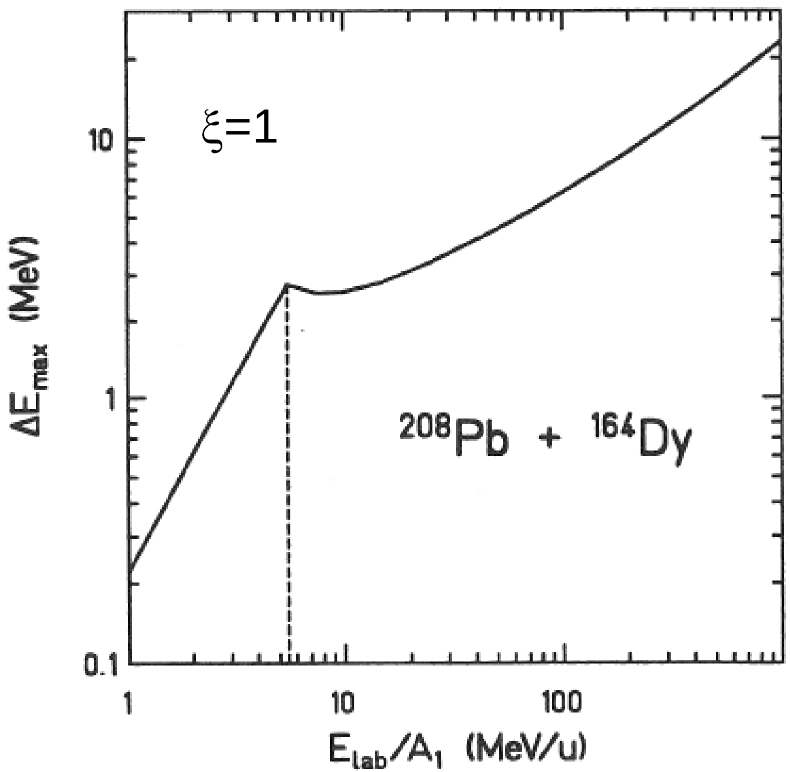
ξ: adiabacity paramater
sometimes also $\xi(\theta)$ with $D(\theta)$ instead of a

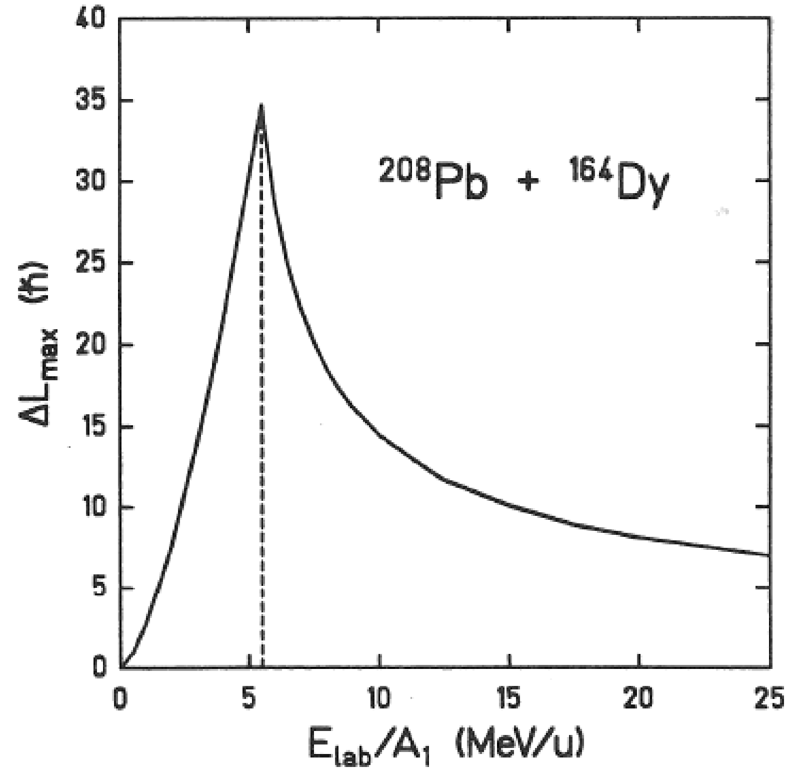
$$\Rightarrow \Delta E_{max}(\xi = 1) = \frac{\hbar v_\infty}{a}$$

Limitation in the excitation energy ΔE for single-step excitations in particular for low-energy reactions (v<c)

Coulomb excitation – first conclusions

Maximal transferable excitation energy and spin in heavy-ion collisions





$$\Delta E_{max}(\xi = 1) = \frac{\hbar c}{a} \frac{v_i}{c} \quad \text{for } v_i/c << 1$$

$$L_{max} = \frac{Z_1 e^2 Q_{rot}}{\hbar v_\infty D^2}(1 - \cos\theta_{cm})$$

$$\Delta E_{max}(\xi = 1) = \frac{\hbar c}{a} \beta\gamma$$

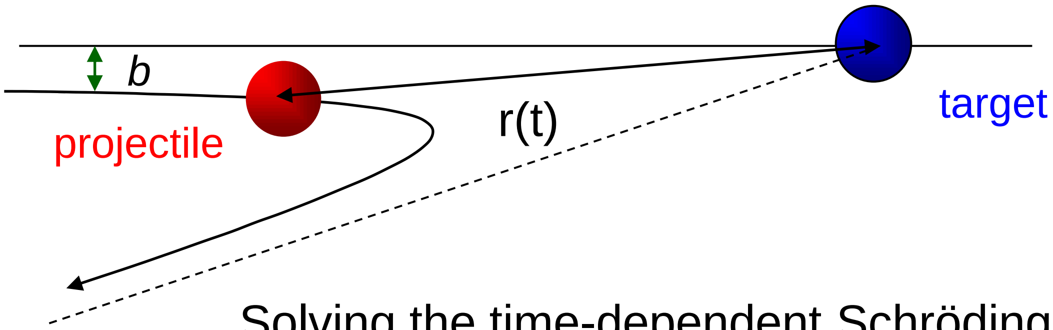
$$L_{max} = \hbar\eta\cot\left(\frac{\theta_{gr}}{2}\right) \quad E_{CM} > V_c$$

Summary I

- Coulomb excitation is a purely electro-magnetic excitation process of nuclear states due to the Coulomb field of two colliding nuclei.
- Coulomb excitation is a very precise tool to measure the collectivity of nuclear excitations and in particular nuclear shapes.
- Coulomb excitation appears in all nuclear reactions (at least in the incoming channel) and can lead to doorway states for other excitations.
- Pure electro-magnetic interaction (which can be readily calculated without the knowledge of optical potentials etc.) requires “safe” distance between the partners at all times.

Transition rates and cross sections in Coulomb excitation

Coulomb excitation theory - the general approach



$r(w) = a\ (\varepsilon \sinh w + 1)$

$t(w) = a/v_{\infty}\ (\varepsilon \cosh w + w)$

$a = Z_p Z_t e^2 E^{-1}$

Solving the time-dependent Schrödinger equation:

$$i\hbar\, d\psi(t)/dt = [H_P + H_T + V(r(t))]\ \psi(t)$$

with $H_{P/T}$ being the free Hamiltonian of the projectile/target nucleus and $V(t)$ being the time-dependent electromagnetic interaction (remark: often only target or projectil excitation are treated)

Expanding $\psi(t) = \sum_n a_n(t)\ \phi_n$ with ϕ_n as the eigenstates of $H_{P/T}$ leads to a set of coupled equations for the time-dependent excitation amplitudes $a_n(t)$

$$i\hbar\, da_n(t)/dt = \sum_m \langle \phi_n | V(t) |\ \phi_m \rangle \exp[i/\hbar\ (E_n - E_m)\ t]\ a_m(t)$$

The transition amplitude b_{nm} are calculated by the (action) integral

$$b_{nm} = i\hbar^{-1} \int \langle a_n \phi_n | V(t) |\ a_m \phi_m \rangle \exp[i/\hbar\ (E_n - E_m)\ t]\ dt$$

Finally leading to the excitation probability

$$P(I_n \rightarrow I_m) = (2I_n + 1)^{-1} b_{nm}^{\ 2}$$

Coulomb excitation theory - the general approach

The coupled equations for $a_n(t)$ are usually solved by a multipole expansion of the electromagnetic interaction V(r(t))

$V_{P\text{-}T}(r) = Z_T Z_P e^2/r$	monopole-monopole (Rutherford) term
$+ \sum_{\lambda\mu} V_P(E\lambda,\mu)$	electric multipole-monopole target excitation,
$+ \sum_{\lambda\mu} V_T(E\lambda,\mu)$	electric multipole-monopole project. excitation,
$+ \sum_{\lambda\mu} V_P(M\lambda,\mu)$ $+ \sum_{\lambda\mu} V_T(M\lambda,\mu)$	magnetic multipole project./target excitation (but small at low v/c)
$+ O(\sigma\lambda,\sigma'\lambda'>0)$	higher order multipole-multipole terms (small)

$$V_{P/T}(E\lambda,\mu) = (-1)^\mu Z_{T/P} e\, 4\pi/(2\lambda+1)\, r^{-(\lambda+1)} Y_{\lambda\mu}(\theta,\phi) \cdot M_{P/T}(E\lambda,\mu)$$

$$V_{P/T}(M\lambda,\mu) = (-1)^\mu Z_{T/P} e\, 4\pi/(2\lambda+1)\, i/c\lambda\, r^{-(\lambda+1)} dr/dt L Y_{\lambda,\mu}(\theta,\phi) \cdot M_{P/T}(M\lambda,\mu)$$

electric multipole moment:

$$M(E\lambda,\mu) = \int \rho(r')\, r'^{\lambda}\, Y_{\lambda\mu}(r')\, d^3r'$$

magnetic multipole moment:

$$M(M\lambda,\mu) = -i/c(\lambda+1) \int j(r')\, r'^{l}\, (ir\times\nabla) Y_{\lambda,\mu}(r')\, d^3r'$$

➔ Coulomb excitation cross section is sensitive to electric multipole moments of all orders, while angular correlations give also access to magnetic moments

Transition rates in the Coulomb excitation process

- **1st order perturbation theory**

➔ Transition probability for multipolarity λ

$$P^{(1)}_{i\to f}(\vartheta,\xi)=|\chi^{(\lambda)}_{i\to f}(\vartheta,\xi)|^2=|\chi^{(\lambda)}_{i\to f}|^2\, R^2_\lambda(\vartheta,\xi)$$

$$\chi^{\lambda}_{i\to f}=\frac{\sqrt{16\pi}(\lambda-1)!}{(2\lambda+1)!!}\left(\frac{Z_{T/P}e}{\hbar v_i}\right)\frac{\langle i|M(E\lambda)|f\rangle}{a^{\lambda}\sqrt{2I_i+1}}$$

Strength parameter

$$R^2_\lambda(\vartheta,\xi)=\sum_\mu |R_{\lambda\mu}(\vartheta,\xi)|^2$$

Orbital integrals

$$\xi=\xi_{if}=\frac{Z_1Z_2e^2}{\hbar}\left(\frac{1}{v_f}-\frac{1}{v_i}\right)$$

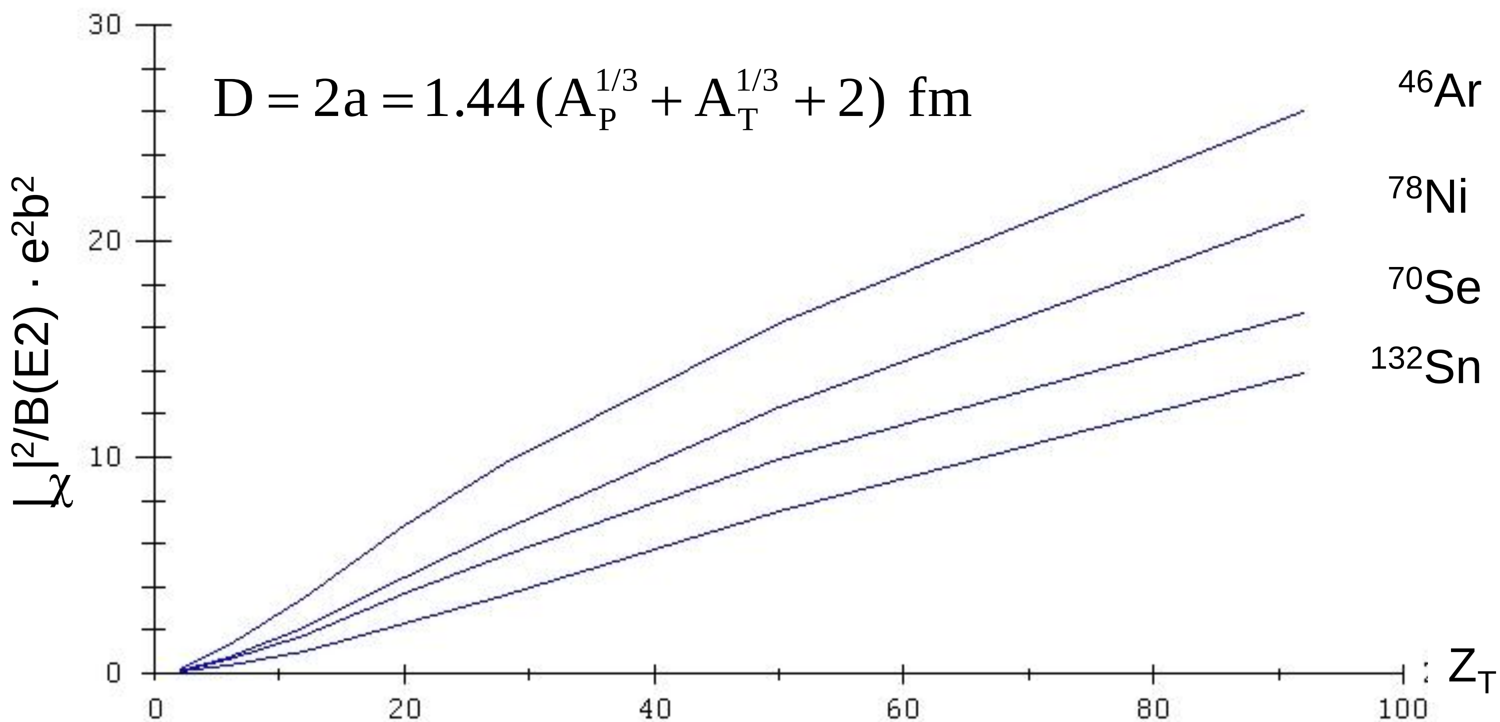
Adiabacity parameter

applicable if only one state is excited, e.g. $0^+\to 2^+$ excitation, and for small interaction strength $\chi^{(\lambda)}$, e.g. semi magic nuclei

Strength parameter χ^{E2} as function of (Z_p, Z_T)

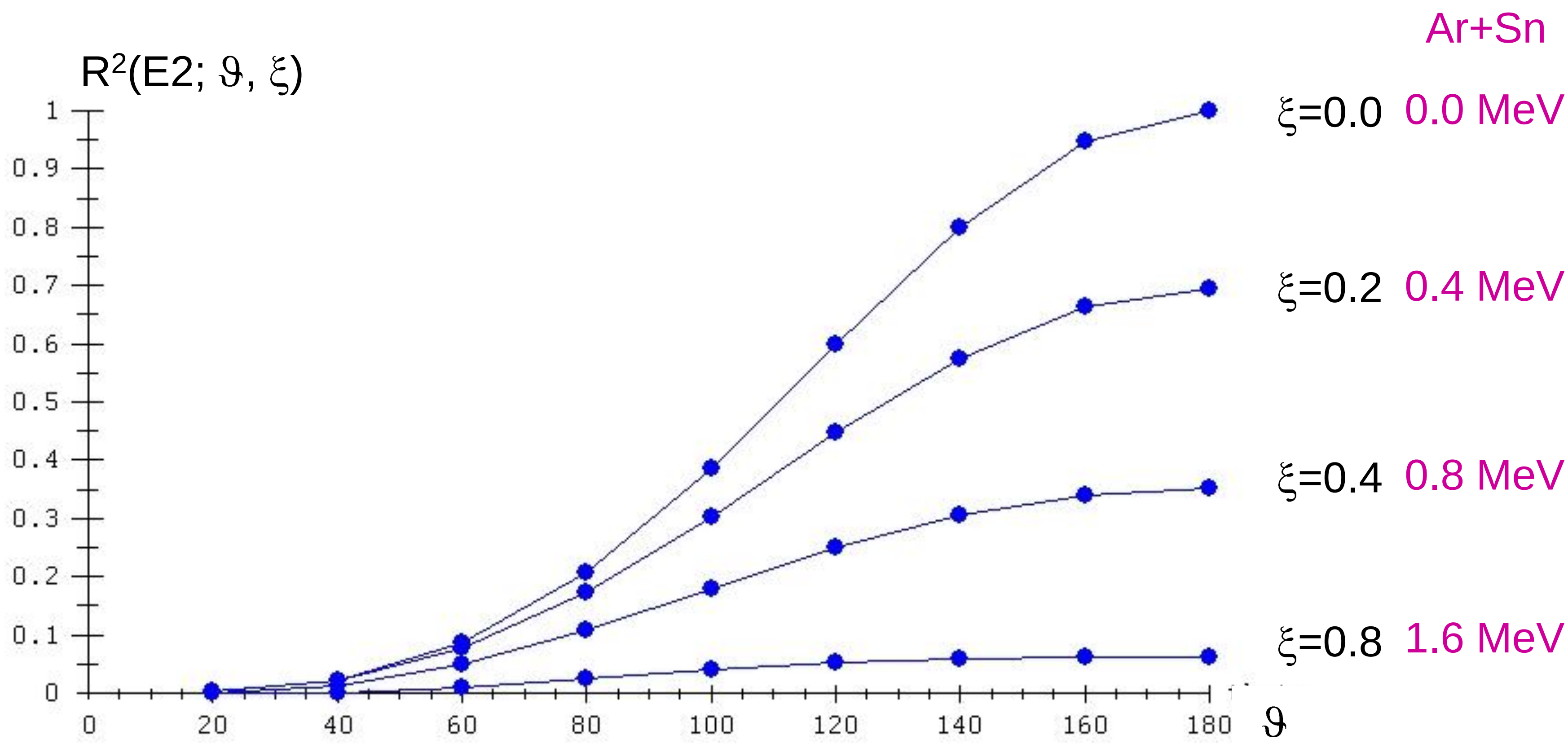
$$|c^{E2}_{i\to f}|^2 = \frac{16\pi}{15^2} \frac{Z^2_{T/P} e^2}{\hbar^2 \, v_i v_f} \frac{B(E2;0^+ \to 2^+)}{(D/2)^4}$$

$$P^{E2}_{i\to f}(J = 180^\circ, x = 0) = |c^{(E2)}_{i\to f}|^2$$



Orbital integrals R(E2) as function of ϑ and ξ

$$R_\lambda^2(\vartheta,\xi) = \sum_\mu |R_{\lambda\mu}(\vartheta,\xi)|^2 \qquad \xi = \xi_{if} = \frac{Z_1 Z_2 e^2}{\hbar}\left(\frac{1}{v_f} - \frac{1}{v_i}\right)$$



Cross section for Coulomb excitation

Differential and total cross sections

$$ds = \tfrac{1}{4} a^2 \sin^{-4}(J/2)\, dW \sum_{\substack{s=E,M \\ l=1,\infty}} | c_{i\to f}^{sl} |^2 R_l^2(J,x)$$

Rutherford

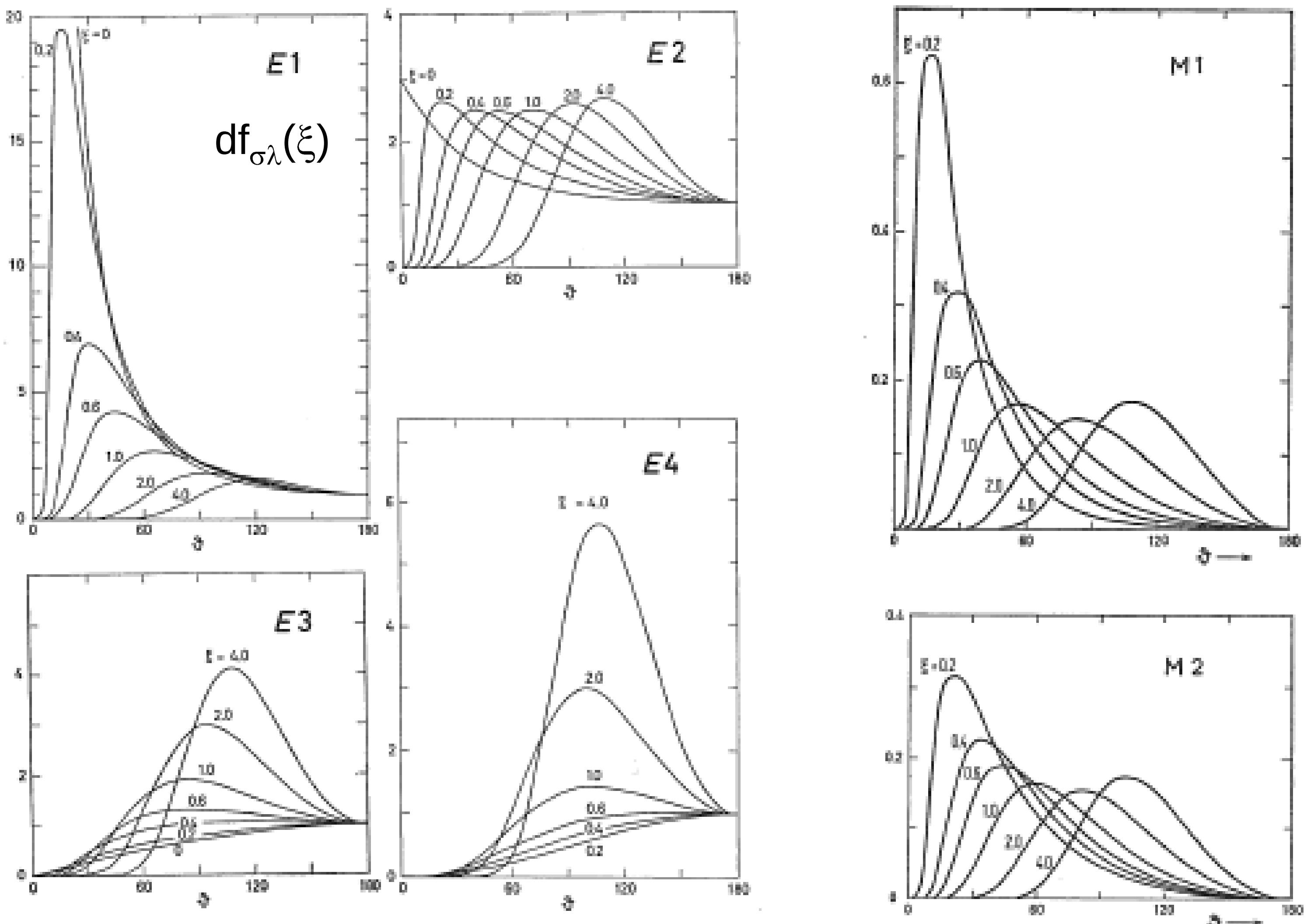
P(σλ)

$$= \left(\frac{Z_1 e}{\hbar v}\right)^2 \sum_{\substack{s=E,M \\ l=1,\infty}} a^{-2(l-1)} B(sl; I_i \to I_f)\, df_{sl}(J,x)$$

$$\sigma = \sum_{\substack{\sigma=E,M \\ \lambda=1,\infty}} \sigma_{\sigma\lambda} = a^2 \sum_{\substack{\sigma=E,M \\ \lambda=1,\infty}} | \chi_{i\to f}^{\sigma\lambda} |^2 \frac{|(2\lambda+1)!!|^2}{16\pi[(\lambda-1)!]^2} f_{\sigma\lambda}(\xi)$$

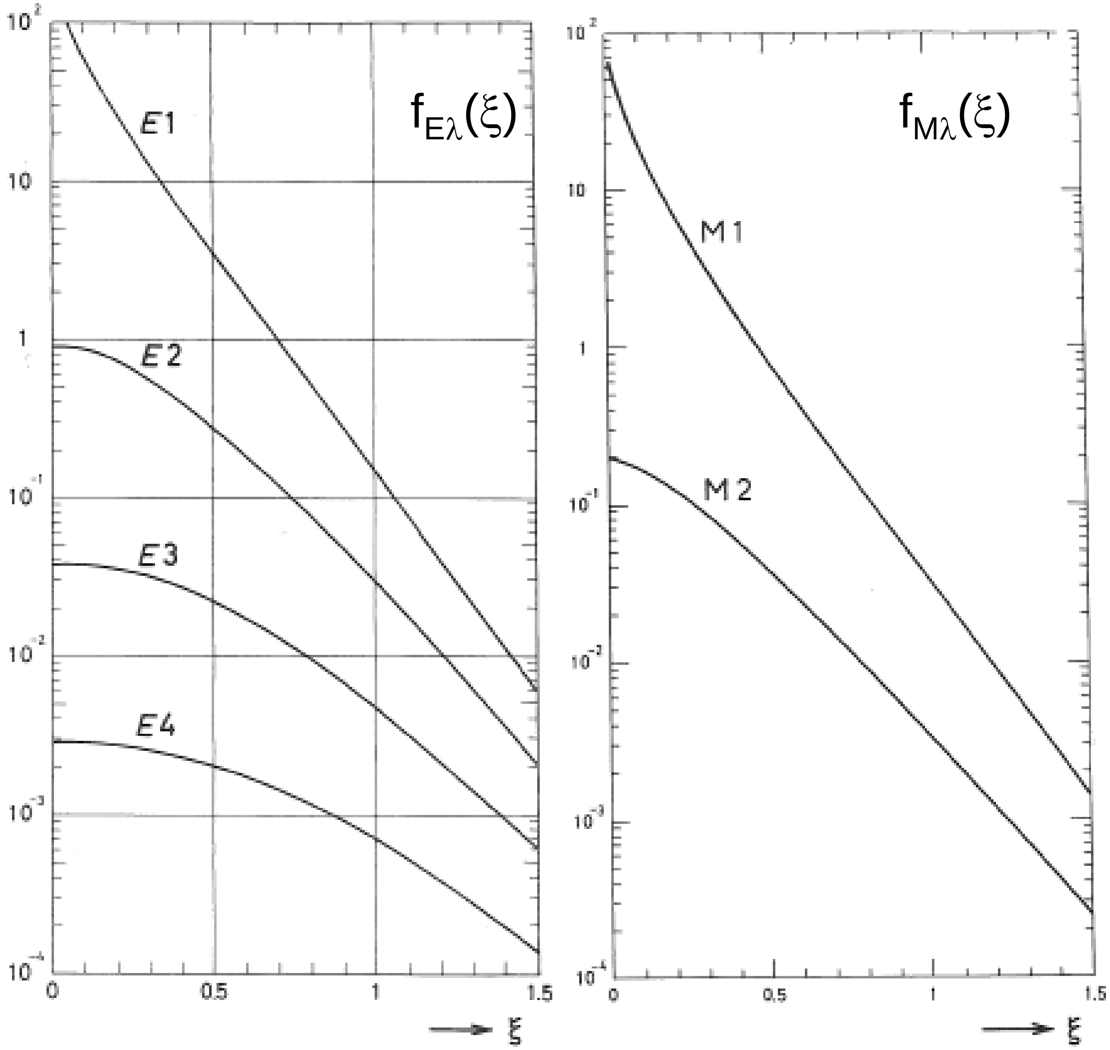
$$= \left(\frac{Z_1 e}{\hbar v}\right)^2 \sum_{\substack{\sigma=E,M \\ \lambda=1,\infty}} a^{-2(\lambda+1)} B(\sigma\lambda; I_i \to I_f)\, f_{\sigma\lambda}(\xi)$$

Angular distribution functions for different multipolarities



$$d\sigma_{\sigma\lambda}/d\Omega = \left(\frac{Z_1 e}{\hbar v}\right)^2 a^{-2(\lambda-1)}\, B(\sigma\lambda; I_i \rightarrow I_f)\, df_{\sigma\lambda}(\xi)$$

Total cross sections for different multipolarities



B(σλ) values for single particle like transitions (W.u.):

$B_{sp}(\lambda) = (2\lambda+1)\, 9e^2/4\pi(3+\lambda)^{-2}\, R^{2\lambda} \times 10(\hbar c/M_pR_0)^2$

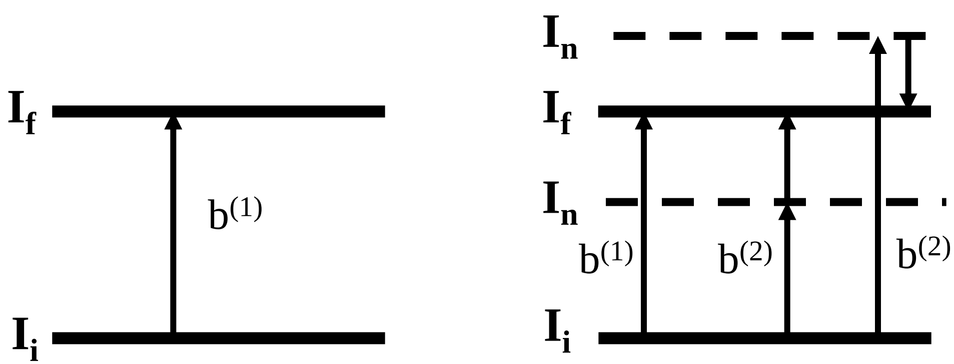
$B(\sigma\lambda)\ [e^2b^\lambda]$	^{208}Pb
E1: $6.45\ 10^{-4}\ A^{2/3}$	$2.3\ 10^{-2}$
E2: $5.94\ 10^{-6}\ A^{4/3}$	$7.3\ 10^{-3}$
E3: $5.94\ 10^{-8}\ A^{2}$	$2.6\ 10^{-3}$
E4: $6.28\ 10^{-10}A^{8/3}$	$9.5\ 10^{-4}$
M1: 1.79	
M2: $0.0594\ A^{2/3}$	2.08

$$\sigma_{\sigma\lambda} = a^2 \mid \chi^{\sigma\lambda}_{i\to f} \mid^2 \frac{|(2\lambda+1)!!|^2}{16\pi[(\lambda-1)!]^2}\, f_{\sigma\lambda}(\xi) = \left(\frac{Z_1 e}{\hbar v}\right)^2 a^{-2(\lambda+1)} B(\sigma\lambda; I_i \to I_f)\, f_{\sigma\lambda}(\xi)$$

Transition rates in the Coulomb excitation process

- **Second order perturbation theory**
  becomes necessary if several states can be excited from the ground state or when multiple excitations are possible i.e. for larger excitation probabilities
  ➔ 2nd order transition probability for multipolarity λ

$$P^{(2)}_{i\to f}(\vartheta,\xi) = (2I_i+1)^{-1}\sum_{m_i m_f} |b^{(2)}_{if}|^2 \quad \text{with } b^{(2)}_{if} = b^{(1)}_{if} + \sum_n b_{inf}$$



1st order:

$$b^{(1)} \propto \langle I_f \| M(E2) \| I_i \rangle$$

2nd order:

$$b^{(2)} \propto \sum_j \langle I_f \| M(E2) \| I_j \rangle \langle I_j \| M(E2) \| I_i \rangle$$

Application to double-step (E2) excitations

- Double-step excitations are important if $\chi_{if} << \chi_{in}\,\chi_{nf} \rightarrow P^{(22)} > P^{(12)}$

➢ 0^+ states can only be excited via an intermediate 2^+ state ($\chi_{if} = 0$)

➔ $P^{(2)} = |\chi_{0\rightarrow 2}|^2\,|\chi_{2\rightarrow 0}|^2\,\pi_0(\theta,s,\xi)$ with $\pi_0(\theta,s,\xi) = 25/4\,(|R_{20}|^2+|G_{20}|^2)$

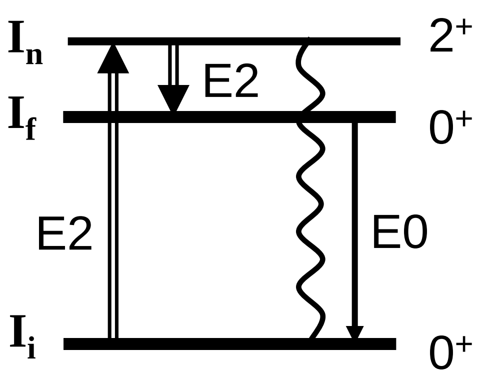
with $\xi = \xi_1 + \xi_2$ and $s = \xi_1/(\xi_1+\xi_2)$

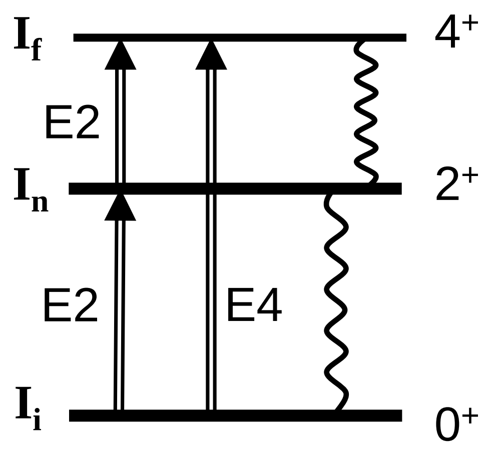
$P^{(2)}\,(\theta=\pi,\ \xi_1=\xi_2\rightarrow 0) \approx 5/4\,|\chi_{0\rightarrow 2}|^2\,|\chi_{2\rightarrow 0}|^2$

➢ 4^+ states are usually excited through a double-step E2 since the direct E4 excitation is small

➔ $P^{(2)} = |\chi_{0\rightarrow 2}|^2\,|\chi_{2\rightarrow 4}|^2\,\pi_4(\theta,s,\xi)$ with $\pi_4(\theta,s,\xi) = 25/4\,(|R_{24}|^2+|G_{24}|^2)$

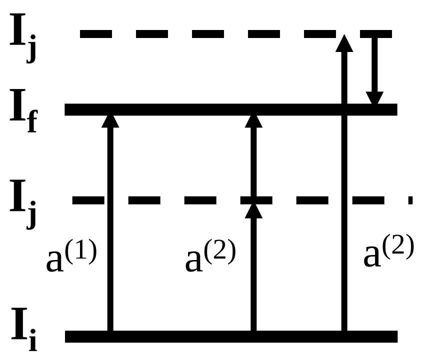
$P^{(2)}\,(\theta=\pi,\ \xi_1=\xi_2\rightarrow 0) \approx 5/14\,|\chi_{0\rightarrow 2}|^2\,|\chi_{2\rightarrow 4}|^2$





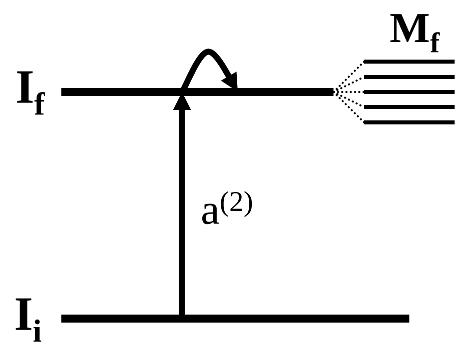
The reorientation effect

- **Specific case of second order perturbation theory** where the „intermediate“ states are the m substates of the state of interest ➔ 2nd order excitation probability for 2^+ state



2nd order:

$$a^{(2)} \propto \sum_j \langle I_f \| \mathrm{M}(E2) \| I_j \rangle \langle I_j \| \mathrm{M}(E2) \| I_i \rangle$$



reorientation effect:

$$a^{(2)} \propto \langle I_f \| \mathrm{M}(E2) \| I_f \rangle \langle I_f \| \mathrm{M}(E2) \| I_i \rangle$$

$$\mathrm{P}^{(2)}_{0\to2}(\vartheta,\xi) = |\chi^{(2)}_{0\to2}|^2 \, \mathrm{R}^2_\lambda(\vartheta,\xi) \left[1 + \chi^{(2)}_{2\to2} c(\vartheta, s=1, \xi)\right]$$

$$\text{with } \chi^{(2)}_{2\to2} = \frac{1}{2}\sqrt{\frac{7}{10}} \frac{\mathrm{e}^2}{\hbar c} \frac{Z_{P/T}}{v_\infty/c} \frac{Q_2}{a^2}$$

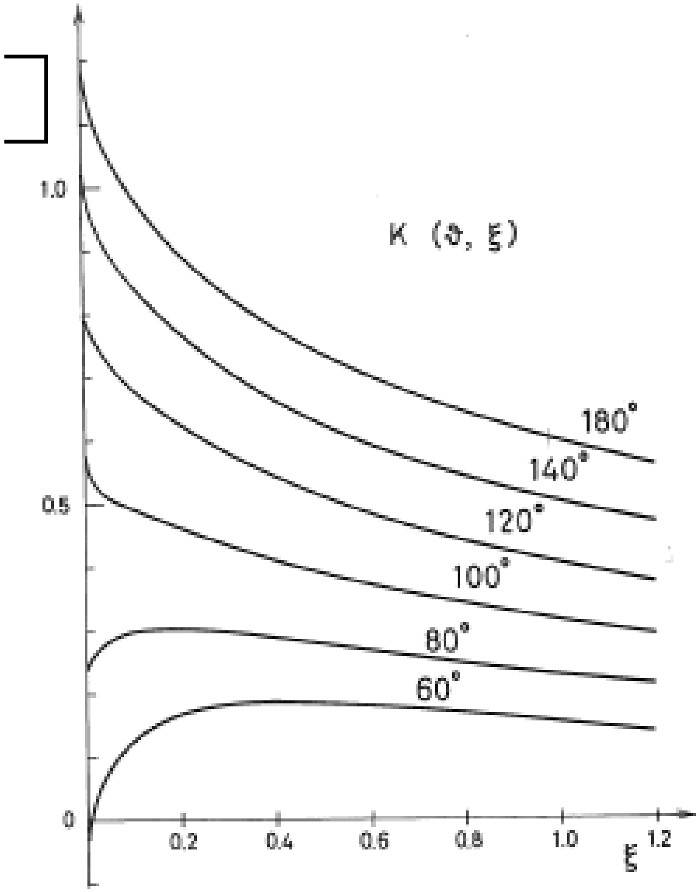
Reorientation strength parameter

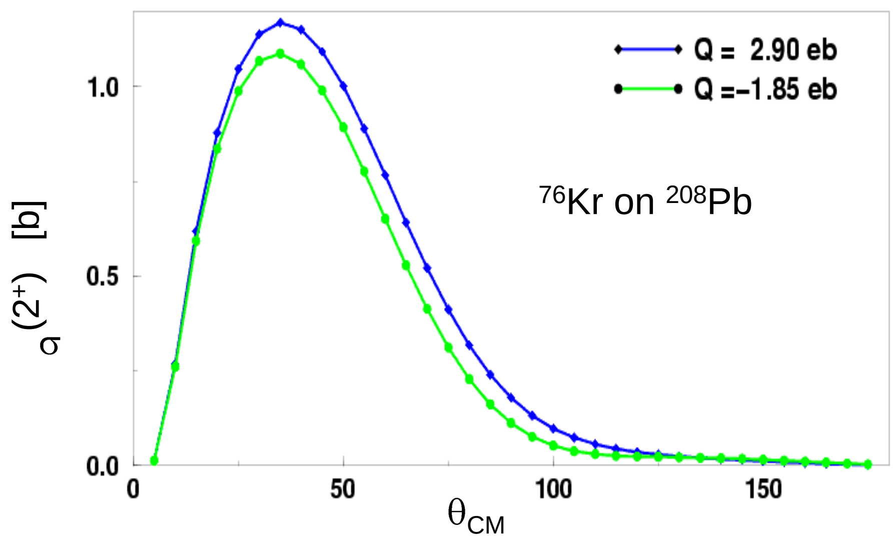
Strength of the reorientation effect

$$P^{(2)}_{0\to 2}(\vartheta,\xi) = P^{(11)}_{0\to 2}(\vartheta,\xi)\left[1 + qK(\vartheta,\xi)\right]$$

$$K(\vartheta,\xi) = \frac{0.5056}{\xi} c(\vartheta,\xi,1)$$

$$q = \frac{A_{T/P}\Delta E_{MeV}}{Z_{P/T}(1 + A_P / A_T)} \sqrt{\frac{7}{2\pi}} \frac{5}{4} Q_2$$





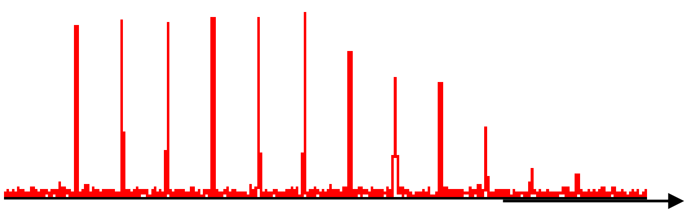
sensitive to diagonal matrix elements
⇒ intrinsic properties of final state:
quadrupole moment including sign

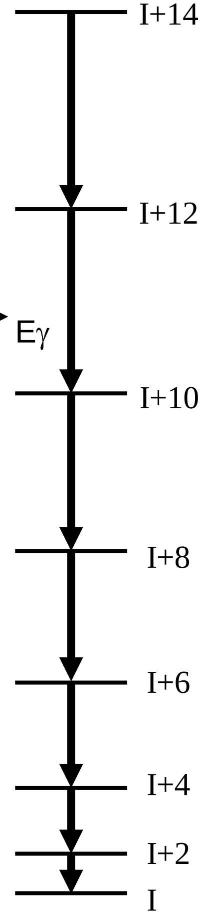
Multi-step Coulomb excitation

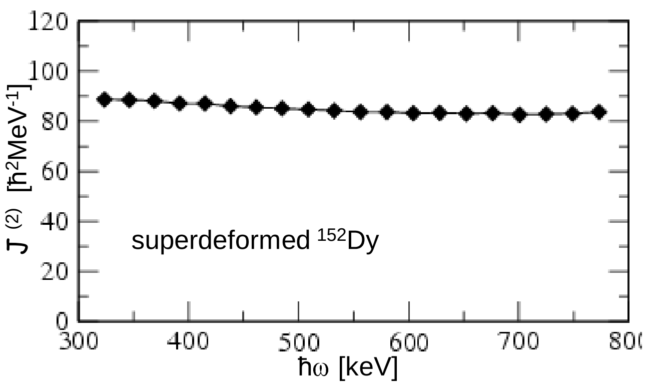
Irfu
cea
saclay

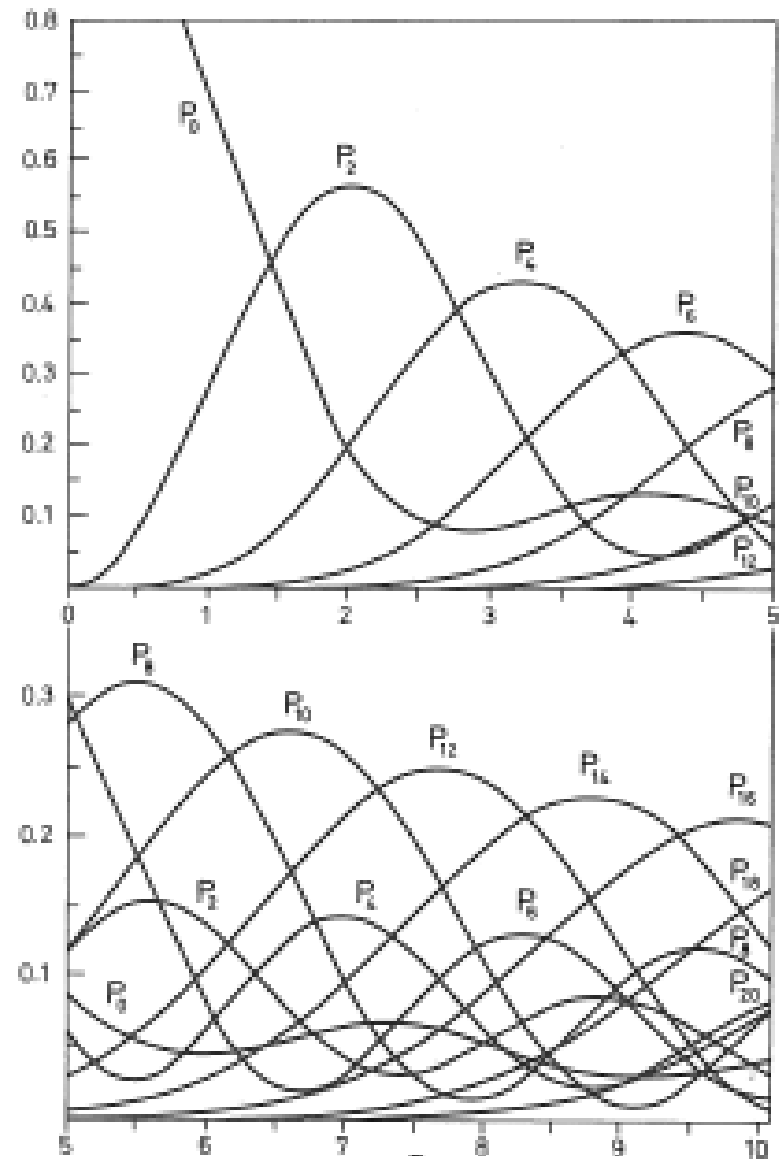
Possible if χ >> 1 (no perturbative treatment)

Example : Rotational band in a strongly deformed nucleus:





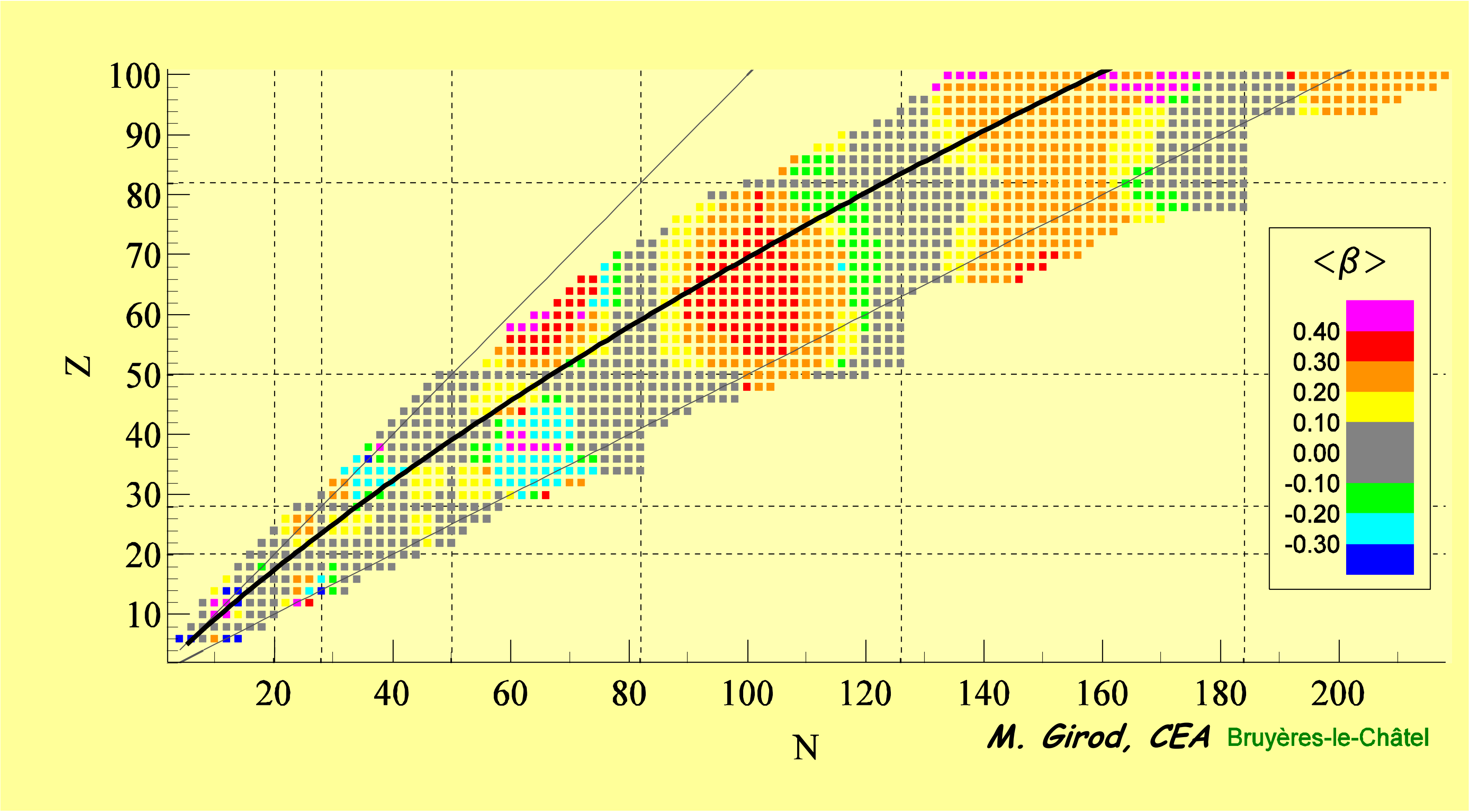




$q = \sqrt{45/16}\ \chi_{0\rightarrow 2}$

Quadrupole deformation of nuclear ground states

Coulomb excitation can, in principal, map the shape of all atomic nuclei:
➔ Quadrupole (and higher-order multipole moments) of I>½ states



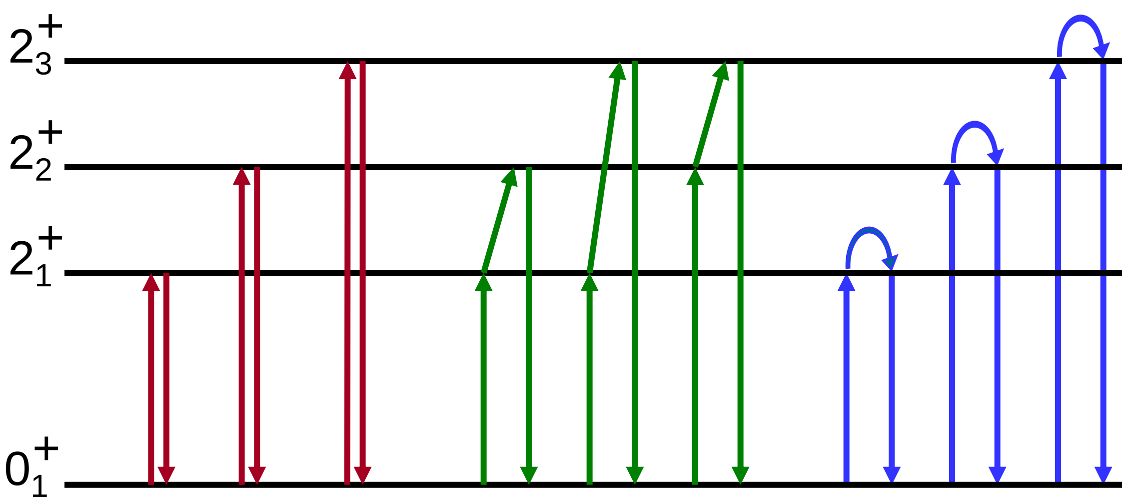
Nuclear deformation and quadrupole sum rules

Model-independent method to determine charge distribution parameters (Q,δ) from a (full) set of E2 matrix elements

$$\mathcal{M}(E2, \mu = 0) = Q \cos\delta$$
$$\mathcal{M}(E2, \mu = \pm 1) = 0$$
$$\mathcal{M}(E2, \mu = \pm 2) = \frac{1}{\sqrt{2}} Q \sin\delta$$

$$\langle s|[E2 \times E2]_0|s\rangle = \frac{1}{\sqrt{5}} Q^2 = \frac{(-1)^{2s}}{\sqrt{2s+1}} \sum_t \langle s||E2||t\rangle\langle t||E2||s\rangle \left\{ \begin{array}{ccc} 2 & 2 & 0 \\ s & s & t \end{array} \right\}$$

$$\langle s|[[E2\times E2]_2 \times E2]_0|s\rangle = -\sqrt{\frac{2}{35}} Q^3 \cos(3\delta) = \frac{1}{2s+1} \sum_{t,u} \langle s||E2||t\rangle\langle t||E2||u\rangle\langle u||E2||s\rangle \left\{ \begin{array}{ccc} 2 & 2 & 2 \\ s & t & u \end{array} \right\}$$



~ Q^2 ~ $Q^3 \cos 3\delta$

→ ground state shape can be determined by a full set of E2 matrix elements i.e. linking the ground state to all collective 2^+ states

Summary II

- Coulomb excitation probability $P(I^{\pi})$ increases with
  increasing strength parameter (χ), i.e. $Z_{P/T}$, $B(\sigma\lambda)$, 1/D, θ_{cm}
  decreasing adiabacity parameter (ξ), i.e. ΔE, a/v_{∞}

- Differential cross sections $d\sigma(\theta)/d\Omega$ show
  varying maxima depending on multipolarity λ and adiabacity parameter ξ
  ➔ allows to distinguish different multipolarities (E2/M1, E2/E4 etc.)

- Total cross section σ_{tot} decreases
  with increasing adiabacity parameter ξ and multipolarity λ
  is generally smaller for magnetic than for electric transitions

- 2[nd] order effects
  lead to "virtual" excitations influencing the real excitation probabilities
  allow to excite 0^+ states and to measure static moments