The mean field approximation to nuclear structure and beyond

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The contents of the three lectures to be delivered at the 2012 edition of the "Ecole Joliot Curie" and devoted to the description of the mean field approximation and techniques beyond it are sketched. The focus will be in the description of phenomena in low energy nuclear structure.

The mean field approximation is the starting point for a quantitative microscopic description of an interacting many body system. The idea is to replace the intricate two body interaction among the system's constituents by an average mean field. When the constituents are fermions, like in the atomic nucleus, the mean field approximation is known under the name of Hartree- Fock (HF) approximation. As in many other many body systems, the effective in medium nuclear interaction has a strong short range attractive component that favors the formation of "Cooper pairs" that are the essential ingredient for phenomena like superfluidity or superconductivity. In those cases, the HF approximation has to be generalized in order to incorporate the concept of quasi-particle leading to the Hartree- Fock- Bogoliubov (HFB) mean field approximation. The mean field HF and HFB have been used for many years in nuclear physics to describe a variety of physical observables ranging from binding energies to the moments of inertia of rotational bands. A lot of expertise has accumulated about the required properties that a good phenomenological in-medium nuclear interaction must have in order to provide a reasonable agreement for many quantities all over the Chart of Nuclide and, as a consequence, a wealth of them are available in the market. At the dawn of the century the general consensus was that both the HFB methodology and the phenomenological interactions to be used were reasonably well understood and therefore, to improve our understanding of the atomic nucleus, other many body effects had to be considered. A typical example is the inclusion of beyond mean field correlations (see below) to improve the description of binding energies and the subsequent impact on other observables that depend on them.

One of the defining characteristics of the mean field approximation when applied to the atomic nucleus is that very often the solution obtained does not preserve the symmetries of the interaction. Typical examples are the breaking of rotational invariance, that leads to the concept of deformed intrinsic states, or the breaking of particle number symmetry associated to nuclear superfluidity and the HFB approximation. Although this spontaneous symmetry breaking is an artifact of the mean field approximation it has the ability to grasp within the mean field framework many relevant correlations like, for instance, the ones leading to the appearance of rotational bands in the low energy spectrum of many nuclei. As a consequence of symmetry breaking the resulting wave function can not be labeled with well defined quantum numbers (particle number, angular momentum, etc). This is irrelevant for some quantities like binding energies or radii if the level of accuracy required is not too high, but it has fundamental consequences for others like transition probabilities where the lack of good quantum numbers makes the concept of "selection rules" (particular choices of the quantum numbers that make the corresponding matrix element to exactly vanish) inapplicable. As a consequence, it is mandatory in some applications to restore the broken symmetries using linear combinations with appropriated weights of a set of wave functions obtained by acting with the corresponding symmetry operation on the HFB intrinsic state. The implementation of these ideas is far from being a trivial task as there remain fundamental issues like how to deal with density dependent effective forces, or technical issues like how to evaluate efficiently the required overlaps. Last but not least, the computational requirements are rather high and often top class supercomputers are required to analyze specific regions of the periodic table with these techniques. This last aspect calls for approximate implementations of symmetry restoration that usually rely on what is called "Gaussian overlap approximation". Here, the main assumption is that the overlap between two different HFB wave functions can be approximated rather well by an analytical expression of the Gaussian type. When this GOA (or its derivations) are implemented, an economical and sound evaluation of many nuclear properties in many nuclei is possible.

The key ingredient of symmetry restoration, namely the use of linear combinations of HFB type wave functions can be also be used to account for long range correlations. The admixture of mean field configurations known as the Generator Coordinate Method (GCM) is the tool of choice to deal with the phenomenon of coexistence where two or more different mean field configurations (for instance, a prolate and an oblate configuration) have a similar energy and feel a strong interaction among them. In the GCM method, the variational principle leads to the Hill-Wheeler equation that determines the amplitudes of the intrinsic configurations in the correlated wave functions. All the quantities entering the HW equation can be evaluated with the same techniques

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used in symmetry restoration and therefore it is not surprising that the same GOA approximation can be used here to reduce the computational needs. The GOA approximation reduces the HW equation, which is a kind of non-local equation to a local approximation known as the "collective Schrodinger equation" that is much easier to solve and its solutions are easier to interpret.

In the three lectures devoted to "mean field and beyond" I will try to cover the main aspects of the techniques discussed above and a tentative schedule could be

- *Lecture 1:* The mean field approximation for fermions: the Hartree-Fock method. Variational method and HF equation. Short range correlations and pairing: the Hartree- Fock- Bogoliubov method. Characterization of HFB wave functions: the Thouless theorem and the Bloch-Messiah theorem. Solution of the HFB and constrained HFB equation with gradient-like methods. Phenomenological interactions: the Skyrme, Gogny and relativistic families. Applications of the HFB method: i) potential energy surfaces 2) High spin physics and the cranking method.

- *Lecture 2:* Spontaneous symmetry breaking in HFB. Restoration of symmetries with projection operators. Intrinsic vs laboratory wave functions. The Variation After Projection (VAP) and Projection After Variation (PAV) methods. Transition probabilities and selection rules. Parity projection as an example. Approximate projection, the "Gaussian overlap approximation" (GOA) and its extensions. Recovering the cranking method and understanding moments of inertia. Recovering the rotational formula for transition probabilities and assessing its applicability.

- *Lecture 3:* Beyond HFB: configuration mixing and the Generator coordinate method. The Hill-Wheeler equation. Evaluation of operator overlaps: The Generalized Wick theorem. The GOA and the derivation of a collective hamiltonian. Where do we stand and what can be expected in the future ?

Concerning the bibliography there are many excellent review articles and textbooks available and here we will just give a sample of the many possible choices and defer a more exhaustive list for the course notes. The first textbook is the well known book by Ring and Schuck [1]. It combines a rigorous treatment of the subject with a wealth of examples and applications. Next, we have the monograph by Blaizot and Ripka [2] that is more theory oriented emphasizing the formals aspects in detriment of the more phenomenological aspects. Finally the review article by Bender, Heenen and Reinhard [3] is a modern account of the theoretical developments taken place in nuclear structure in the last years.

[1] P. Ring and P. Schuck, *The Nuclear Many Body problem* (Springer, Berlin, 1980)
[2] J.-P. Blaizot and G. Ripka, *Quantum Theory of Finite Systems* (MIT Press, Cambridge, Massachusetts, and London, England, 1985).
[3] M. Bender, P.-H. Heenen, and P.-G. Reinhard, Rev. Mod. Phys. **75** (2003) 121.