The nuclear shell model: from single-particle motion to collective effects

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Lecture 1: Nuclear forces and very light nuclei

Before attempting to start a description of the atomic nucleus as a bound system of interacting nucleons, it is paramount to learn about the nature of the nucleon-nucleon interactions. We start from the basic properties of nucleon-nucleon interactions as manifested from free nucleon-nucleon scattering. After a short reminder of the Schrödinger equation describing scattering of two nucleons through a central potential V(r), we show that the properties of nucleon-nucleon scattering can be described by a set of nuclear phase shifts. We discuss these results for the dominant scattering channels and their importance in understanding the nucleon-nucleon interaction as a function of the collision energy in free space. The next step is to derive an analytical form describing the experimental scattering data. We are able to constrain the most general form using general invariance properties from quantum mechanics which the nucleon-nucleon interaction should obey. We stress in particular the importance of a tensor term, which is related to the Yukawa picture using one-pion exchange as one of the decisive elements describing nucleon-nucleon forces, as well as the need of a spin-orbit term. We present an example of one of the early realistic nucleon-nucleon forces (meaning the force is able to describe nuclear phase shifts up to $E_{lab} \sim 400$ MeV) as well as the more recent Argonne potentials. These nucleon-nucleon forces describe the extensive experimental data set of nucleon-nucleon phase shifts extremely well as will be illustrated. Moving to very recent work, it is shown that the most important central components of the free nucleon-nucleon interaction can be derived from lattice QCD calculations.

Attempting to describe the very light nuclei such as ^{3}H and ^{3}He starting from realistic nucleon-nucleon interactions, however, leads to underbound nuclei. This shows the need of introducing three-nucleon interaction terms which is illustrated for the nuclei up to mass number A=12 (^{12}C), indicating the need of both two-nucleon and three-nucleon terms to describe these very light nuclei from ab-initio calculations. We close by facing the problems that arise when trying to use these realistic interactions in order to describe the effective interactions inside the nucleus, containing many interacting protons and neutrons.

Lecture 2: The independent-particle shell model and few nucleon correlations

The stability of nuclei and abundances of the elements (BE(A) ~ A), as well as early experimental data on nuclear ground-state spins and magnetic moments in many odd-mass nuclei were all pointing towards the existence of a central potential in which nucleons move, to a large extent, as independent particles. In a first part we discuss the concept of an independent-particle model, showing that a harmonic oscillator potential enlarged with a centrifugal ***l.l*** and spin-orbit ***l*.s** term, is able to describe consistently the proton and neutron numbers corresponding to nuclei with increased stability. We go on showing in some detail the salient features of the shell-model structure (single-particle energy spectra and wave functions, the shell-gaps in the spectra,...).

We then discuss nuclei containing two nucleons outside of a closed-shell system and analyze the energy correction that results from the interaction between these two ‘valence’ nucleons. In order to obtain quantitative results, we start from simple central forces to derive the interaction energy, splitting the degeneracy of the two-nucleon system. A generic property shows up, i.e., the two nucleons (protons or neutrons) preferentially pair up in a state of angular momentum J = 0. We illustrate this important result with examples such as the separation energy of a nucleon from a given nucleus as well as analyzing energy spectra of nuclei with two nucleons outside (or missing) of a closed-shell core.

We move on to make the simple analyses, in which the two nucleons are constrained to one single-particle orbital, more realistic. In general, the two nucleons can occupy more single-particle states and consequently give rise to a number of possible configurations, forming a quantum mechanical basis to solve the nuclear energy eigenvalue problem, starting from a given two-body interaction. We present, in detail, the procedure for a simple nucleus ^{18}O, in which the two valence nucleons can move in the N=2 ($1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$) orbitals. We discuss the results in the case of ^{18}0 and ^{210}Po, pointing out the presence of a generic pairing results that two-nucleon correlations exhibit throughout the nuclear mass table.

Lecture 3: Many-nucleon correlations: collective excitations and symmetries

Starting from the detailed discussion for two-nucleon systems, we generalize the nuclear energy eigenvalue problem to systems with a large number of active protons and/or neutrons moving in a given model space (configuration space). To do so we discuss (i) how to build a basis needed to expand the full wave function, emphasizing the rapid increase in dimension of the eigenvalue problem and the corresponding computational issues, and, (ii) how to handle the nucleon-nucleon interaction acting inside the atomic nucleus. The latter point needs particular attention since one can (i) start from relatively simple schematic interactions (putting forward an analytical form), (ii) start from the nuclear two-body matrix elements, called the empirical effective interaction, which are used as parameters that are fixed by fitting the calculated observables (energy, lifetimes in γ- and β-decay,…) to a large set of the corresponding nuclear data in a given region of the nuclear mass table, or, (iii) start from a realistic interaction (see Lecture 1), however, adapting this force for use inside an atomic nucleus constructing the nuclear G-matrix.

We subsequently go through the various steps in order to apply the nuclear shell model for the rather complex systems of many interacting nucleons (encompassing both protons and/or neutrons). We illustrate the advances that have been made from the early 1p-shell nuclei (1965) towards recent calculations with very big model spaces and many nucleons. In particular we show results for the sd shell and the fp shell. In the latter case, it is shown that structures reminiscent of collective modes of motion (macroscopic) appear.

We make a side-step to show what may be the deeper origin of such collective modes resulting from a purely microscopic shell-model approach.

Finally, we consider nuclei with a single closed shell, such as the N=126 isotones and the Sn isotopes. We point out that the characteristic energy spectra resulting from the shell-model calculations can be understood from an unexpected point of view, emphasizing symmetries connected to the formation of nucleon pairs in long series of isotones (isotopes).

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