SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

BASIC WEAK-DECAY EQUATION

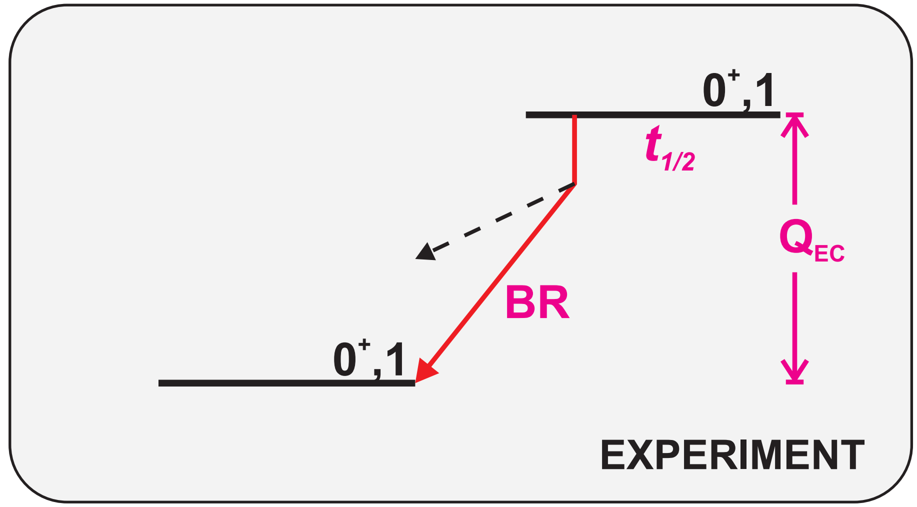
$$ft = \frac{K}{G_V^2 < \quad >^2}$$

f = statistical rate function: $f(Z, Q_{EC})$

t = partial half-life: $f(t_{1/2}, BR)$

G_V = vector coupling constant

$< \quad >$ = Fermi matrix element



INCLUDING RADIATIVE AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS

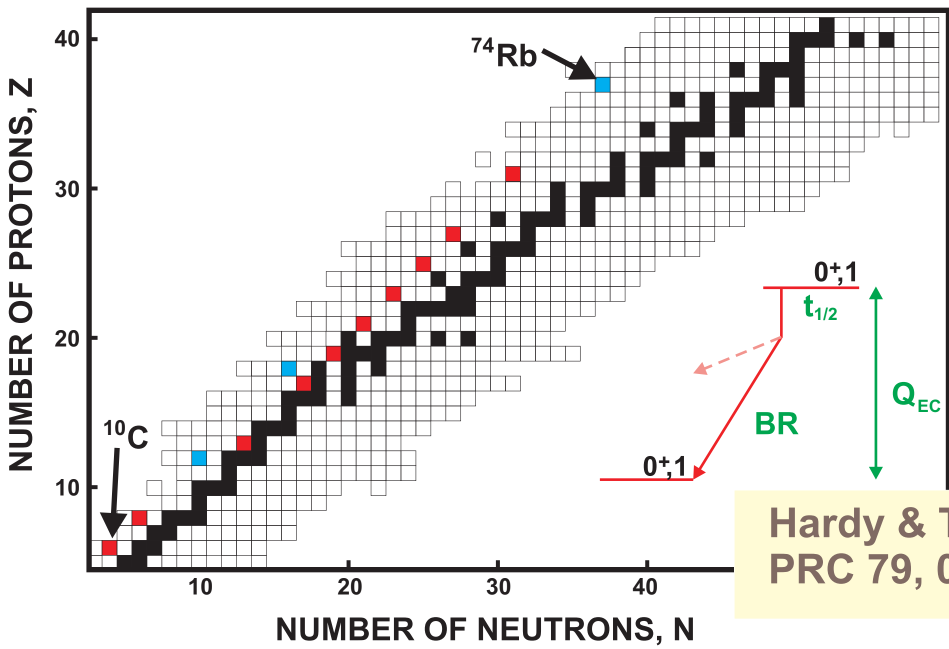
$$\mathcal{F}t = ft\,(1 + {}'_{R})[1 - ({}_{C} - {}_{NS})] = \frac{K}{2G_V^2\,(1 + {}_{R})}$$

$f(Z, Q_{EC})$
~1.5%

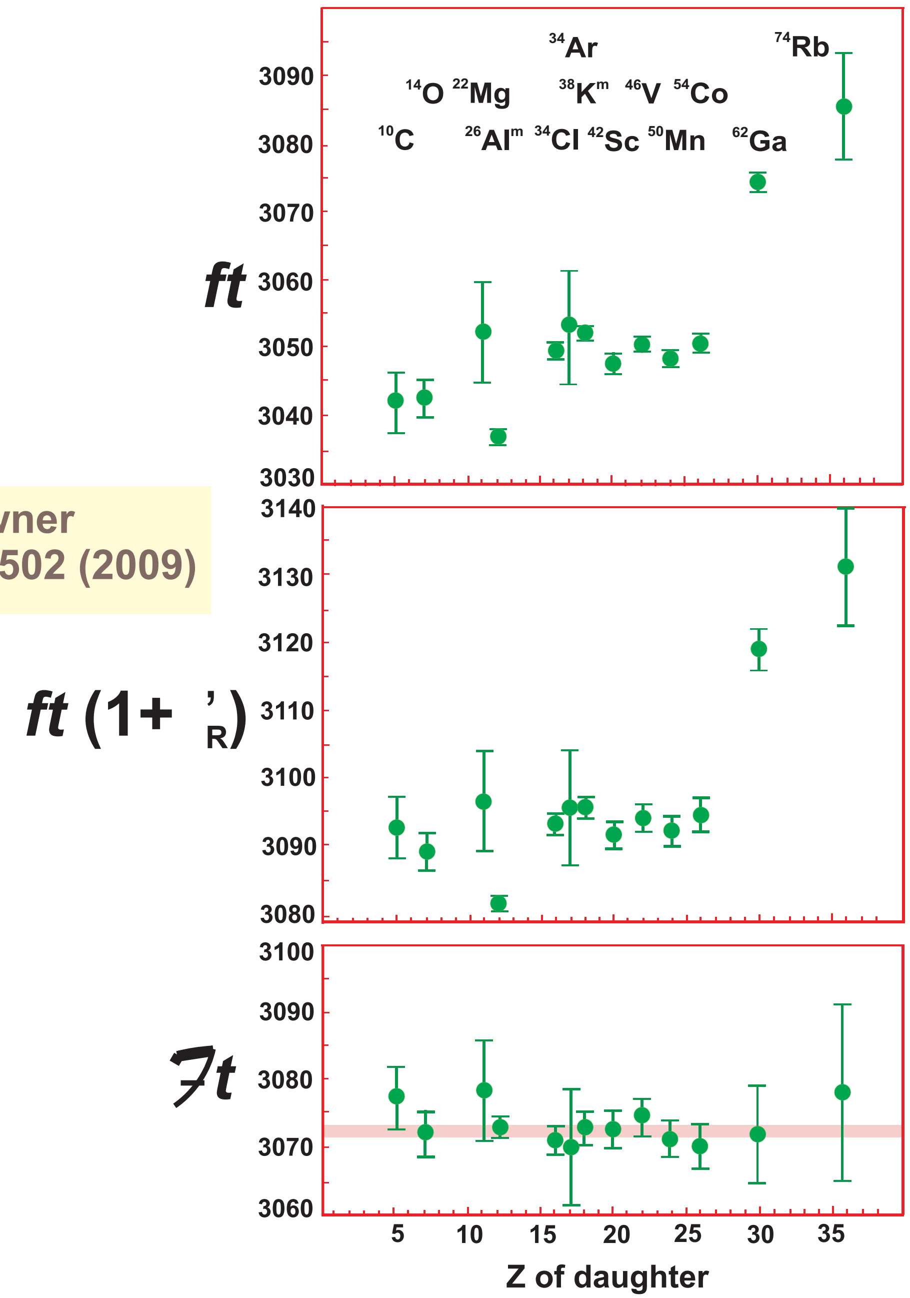
f(nuclear structure)
0.3-1.5%

f(interaction)
~2.4%

WORLD DATA FOR $0^+ \rightarrow 0^+$ DECAY, 2009



Hardy & Towner
PRC 79, 055502 (2009)



- 10 cases with *ft*-values measured to ~0.1% precision; 3 more cases with <0.3% precision.
- ~150 individual measurements with compatible precision

$$\mathcal{F}t = ft\,(1 + {}'_R)[1 - ({}_C - {}_{NS})] = \frac{K}{2G_V^{\,2}\,(1 + {}_R)}$$

THE PATH TO V_{ud}

FROM A SINGLE TRANSITION

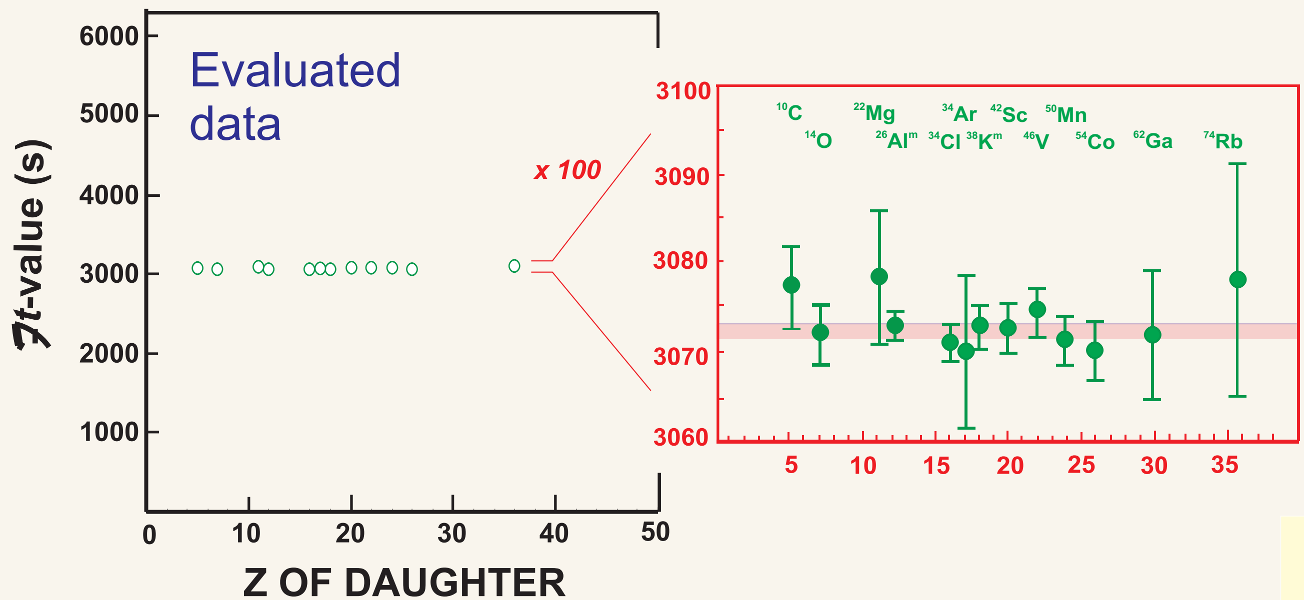
Experimentally determine $G_V^2(1 + \;_R)$

$$\mathcal{F}t = ft\,(1 + \;'_R)[1 - (\;_C - \;_{NS})] = \frac{K}{2G_V^2\,(1 + \;_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

G_V constant to ± 0.013%



$\overline{\mathcal{F}t}$ = 3072.2(8)

$G_V(1+\;_R)^{1/2}/(\hbar c)^3$
= 1.14961(15)
x10^{-5} GeV^{-2}

Hardy & Towner
PRC 79, 055502 (2009)

THE PATH TO V_{ud}

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \quad {}_R)$

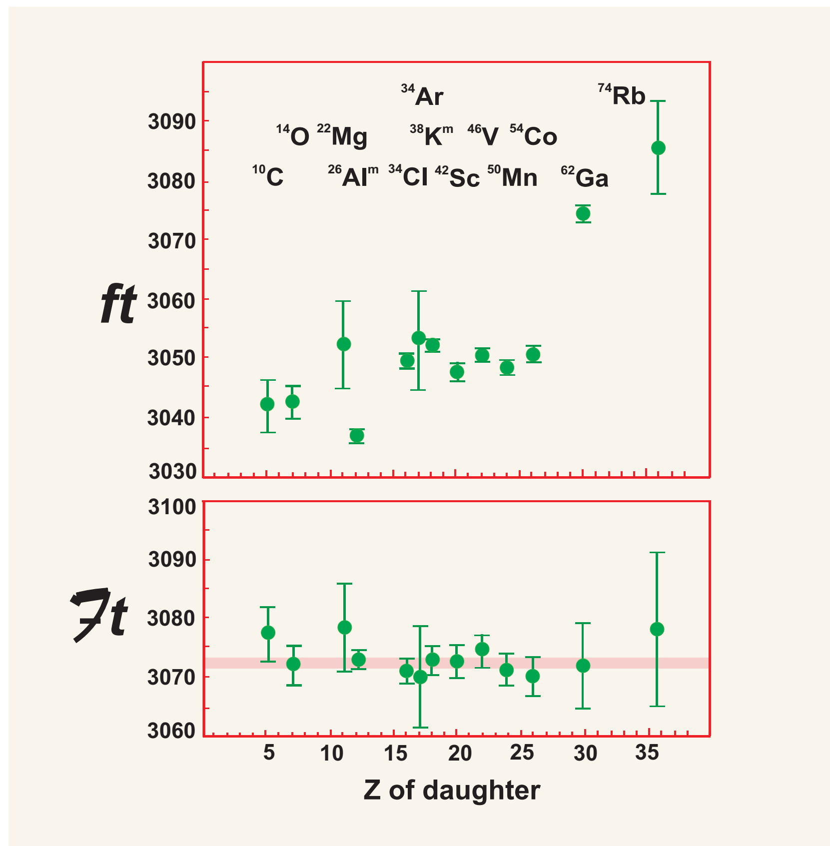
$$\mathcal{F}t = ft\,(1 + \quad {}'_R)[1 - (\quad {}_C - \quad {}_{NS})] = \frac{K}{2G_V^2\,(1 + \quad {}_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms ✔

G_V constant to ± 0.013%



THE PATH TO V_{ud}

FROM A SINGLE TRANSITION

Experimentally determine $G_V^{\,2}(1 + \ _R)$

$$\mathcal{F}t = ft\,(1 + \ '_R)[1 - (\ _C - \ _{NS})] = \frac{K}{2G_V^{\,2}\,(1 + \ _R)}$$

FROM MANY TRANSITIONS

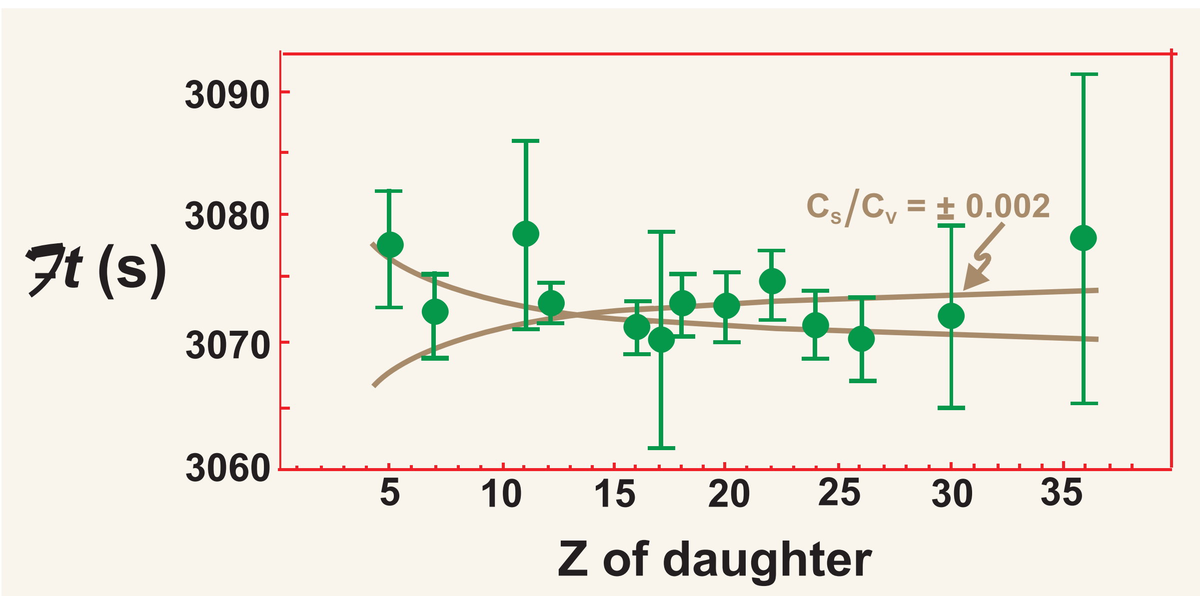
Test Conservation of the Vector current (CVC)

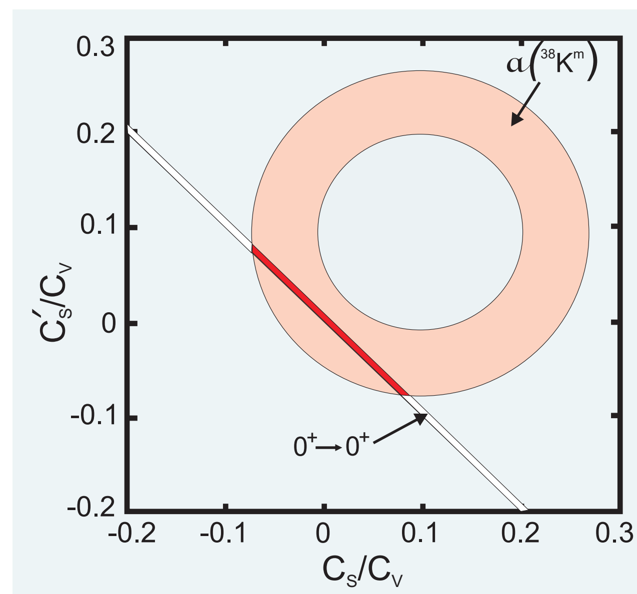
G_V constant to ± 0.013%

Validate correction terms ✔

Test for Scalar current

limit, $C_S/C_V = 0.0011\ (14)$





THE PATH TO V_{ud}

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \quad_R)$

$$\mathcal{F}t = ft\,(1 + \,'_R)[1 - (\;_C - \;_{NS})] = \frac{K}{2G_V^2\,(1 + \;_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms ✔

Test for Scalar current

G_V constant to ± 0.013%

limit, C_S/C_V = 0.0011 (14)

WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

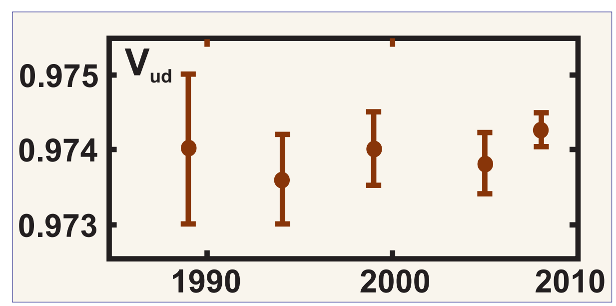
weak eigenstates

mass eigenstates

Obtain precise value of $G_V^2(1 + \quad_R)$

Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G\;^2 = 0.94916 \pm 0.00044$$



THE PATH TO V_{ud}

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + {}_R)$

$$\mathcal{F}t = ft\,(1 + {}'_R)[1 - ({}_C - {}_{NS})] = \frac{K}{2G_V^2\,(1 + {}_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms ✔

Test for Scalar current

G_V constant to ± 0.013%

limit, $C_S/C_V = 0.0011\ (14)$

WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates

mass eigenstates

Obtain precise value of $G_V^2(1 + {}_R)$

Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G^2 = 0.94916 \pm 0.00044$$

Test CKM unitarity

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9999 \pm 0.0006$$

THE PATH TO V_{ud}

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \ _R)$

$$\mathcal{F}t = ft\,(1 + \ '_R)[1 - (\ _C - \ _{NS})] = \frac{K}{2G_V^2\,(1 + \ _R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms ✔

Test for Scalar current

G_V constant to ± 0.013%

limit, C_S/C_V = 0.0011 (14)

WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates

mas eigen

Obtain precise ... of $G_V^2(1 + \ _R)$

Determin...

$V_{ud}^2 = G_V^2/G^2 = 0.94916 \pm 0.00044$

... unitarity

$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9999 \pm 0.0006$

ONLY POSSIBLE IF PRIOR CONDITIONS SATISFIED

T=1/2 SUPERALLOWED BETA DECAY

BASIC WEAK-DECAY EQUATION

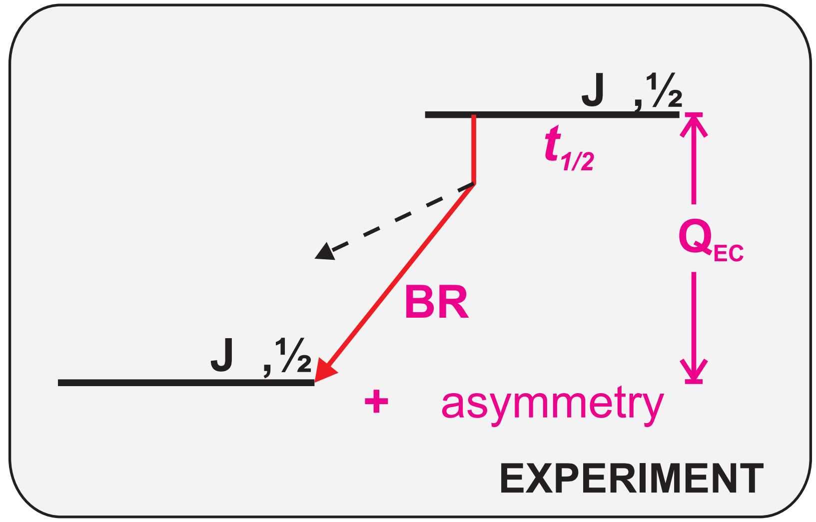
$$ft = \frac{K}{G_V^2\langle\ \ \rangle^2 + G_A^2\langle\ \ \ \ \rangle^2}$$

f = statistical rate function: f(Z, Q_{EC})

t = partial half-life: f($t_{1/2}$, BR)

$G_{V,A}$ = coupling constants

< > = Fermi, Gamow-Teller matrix elements



INCLUDING RADIATIVE CORRECTIONS

$$\mathcal{F}t = ft\,(1 + {}'_R)[1 - ({}_C - {}_{NS})] = \frac{K}{G_V^2\,(1 + {}_R)(1 + {}^2\langle\ \ \ \rangle^2)}$$

= G_A/G_V

NEUTRON DECAY

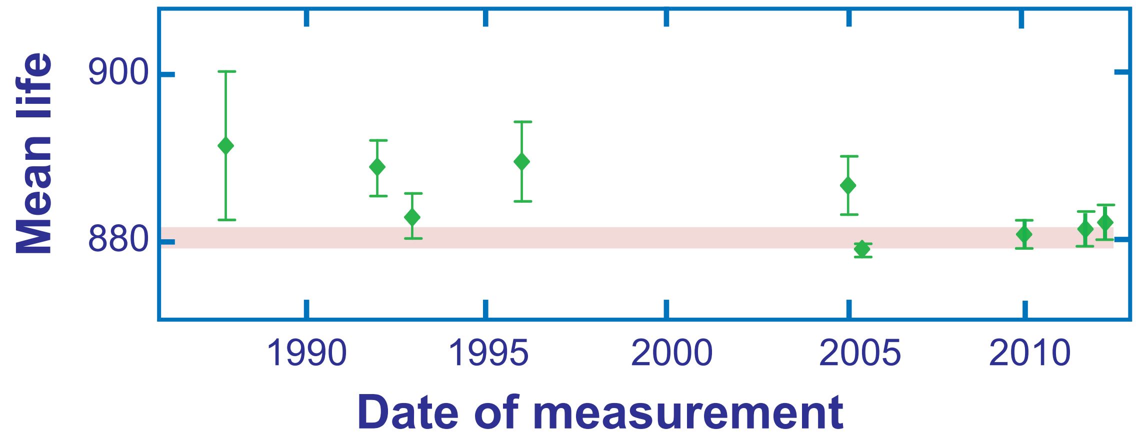
Requires additional experiment:
for example, asymmetry (A)

NEUTRON DECAY DATA 2012

Mean life:

= 880.4 ± 1.2 s

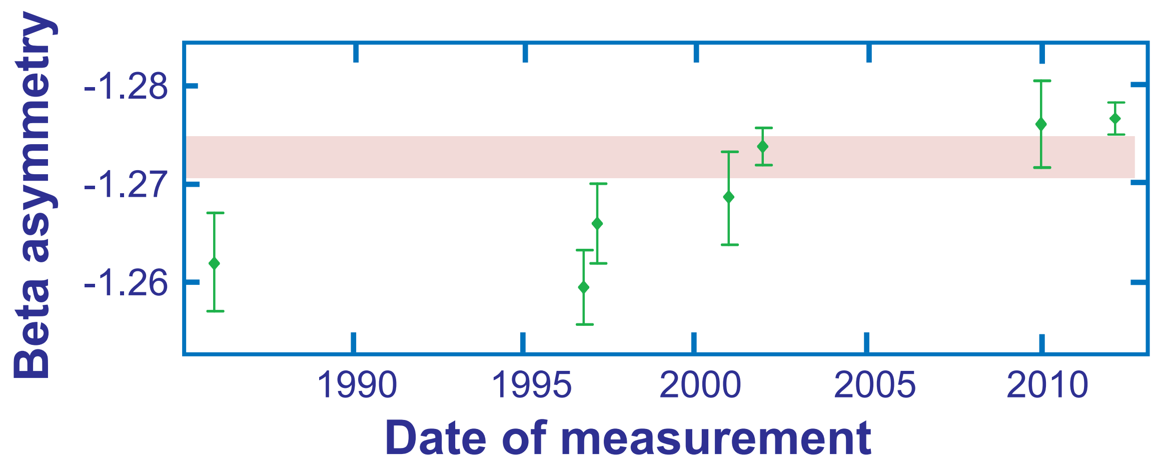
^{2}/N = 3.5



asymmetry:

= -1.2728 ± 0.0022

^{2}/N = 4.6



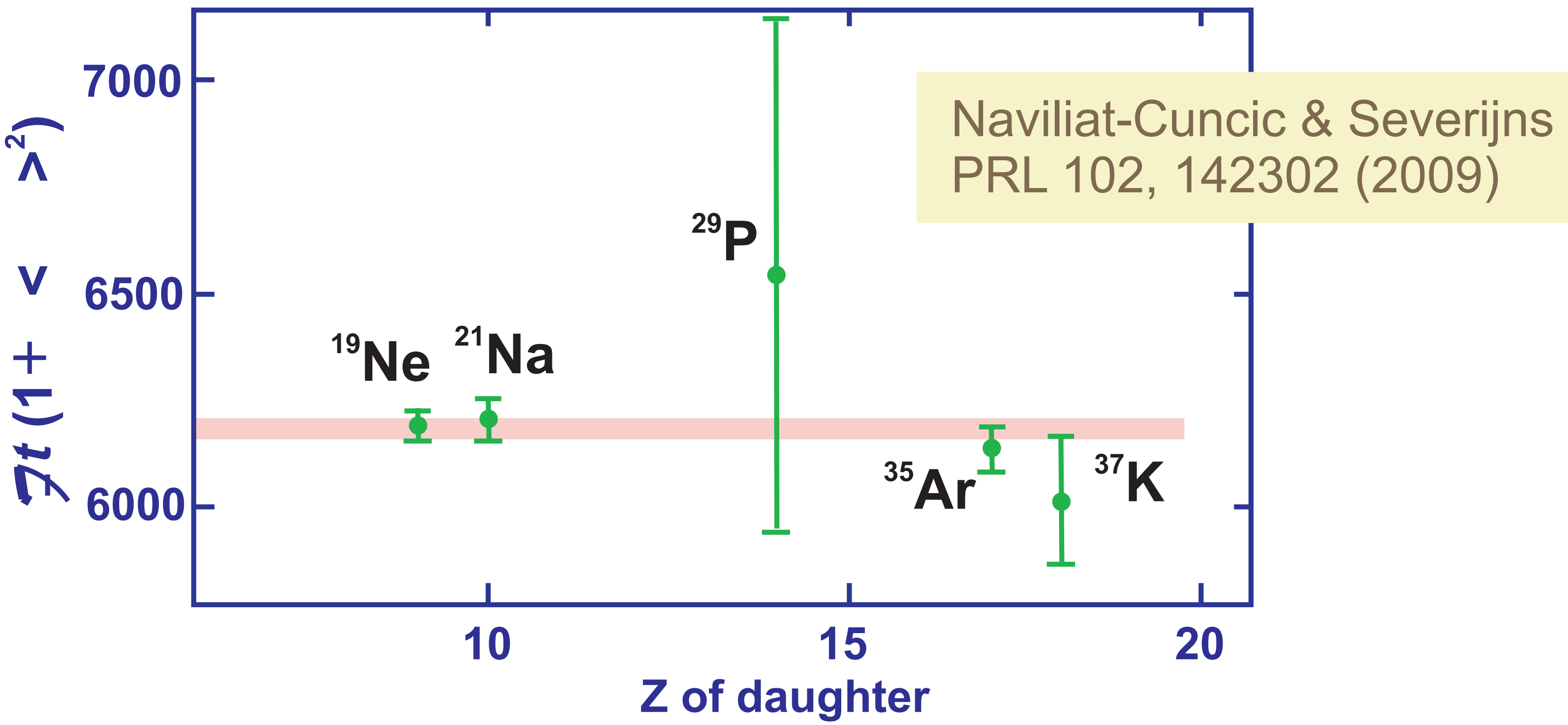
V_{ud} = 0.9754 ± 0.0016

nuclear $0^{+} \rightarrow 0^{+}$

V_{ud} = 0.9743 ± 0.0002

NUCLEAR T=1/2 MIRROR DECAY DATA 2009

$$\mathcal{F}t = ft\,(1 + {}'_{R})[1 - ({}_{C} - {}_{NS})] = \frac{K}{G_V^2\,(1 + {}_{R})(1 + \ < \ >^2)}$$



$V_{ud} = 0.9719 \pm 0.0017$

nuclear $0^+ \rightarrow 0^+$

$V_{ud} = 0.9743 \pm 0.0002$

PION BETA DECAY

Decay process:

$$^{+} \longrightarrow \;^{0}\, e^{+} \quad _{e}$$

$$0^{-},1 \longrightarrow 0^{-},1$$

Experimental data:

$$= 2.6033 \pm 0.0005 \times 10^{-8}\ \mathrm{s}$$ (PDG 2009)

$$\mathrm{BR} = 1.036 \pm 0.007 \times 10^{-8}$$

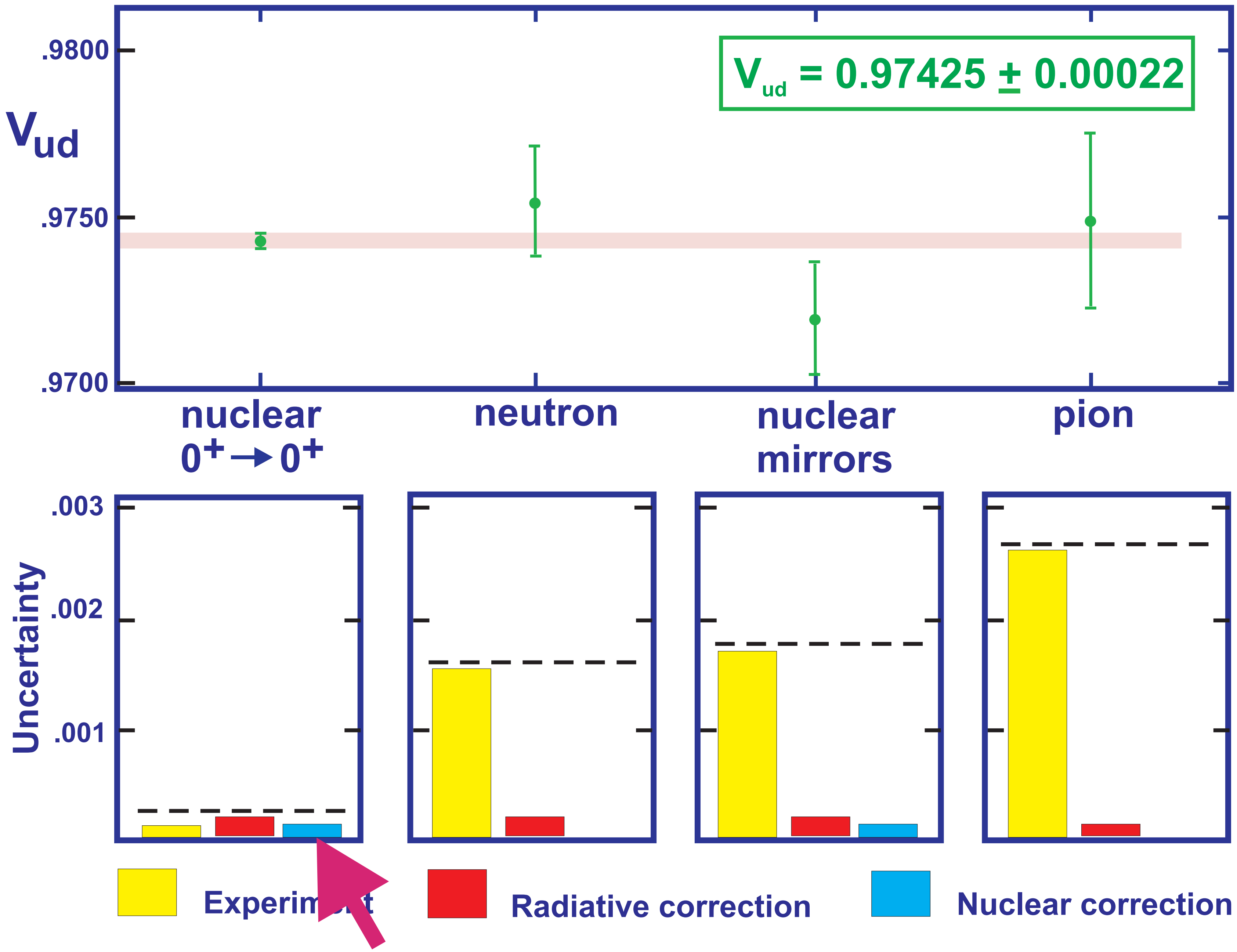
Pocanic *et al*,
PRL 93, 181803 (2004)

Result:

$$\boxed{V_{ud} = 0.9749 \pm 0.0026}$$

nuclear $0^{+} \rightarrow 0^{+}$
$V_{ud} = 0.9743 \pm 0.0002$

CURRENT STATUS OF V_{ud} – 2009



CALCULATED CORRECTIONS TO $0^+ \rightarrow 0^+$ DECAYS

$$\mathcal{F}t = ft\,(1 + {}'_R)[1 - ({}_C - {}_{NS})] = \frac{K}{2G_V^2\,(1 + {}_R)}$$

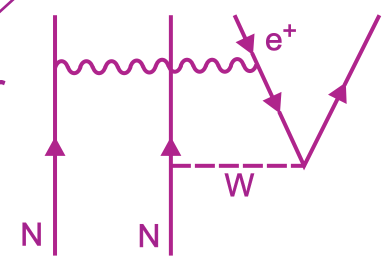
1. Radiative corrections

$$'_R = \frac{}{2}\,[g(E_m) + {}_2 + {}_3 + \ldots]$$

$$_R = \frac{}{2}\,[4\ln(m_Z/m_p) + \ln(m_p/m_A) + 2C_{Born} + \ldots]$$

$_{NS}$

Order- axial-vector universal photonic contributions



Dependent on nuclear structure

2. Isospin symmetry-breaking corrections

$_C$ Charge-dependent mismatch between parent and daughter analog states (members of the same isospin triplet).

ISOSPIN SYMMETRY BREAKING CORRECTIONS

$$C = C1 + C2$$

C1	C2
Difference in configuration mixing between parent and daughter.	Mismatch in radial wave function between parent and daughter.
✓ • Shell-model calculation with well-established two-body matrix elements.	✓ • Full-parentage Wood-Saxon wave function matched to known binding energy and charge radius from electron scattering.
✓ • Charge dependence tuned to known single-particle energies and to measured IMME coefficients.	✓ • Compared with Hartree-Fock calculation matched to known binding energy.
✓ • Results also adjusted to measured 0^+ state energies.	✓ • Core states included based on measured spectroscopic factors.
0.01 – 0.3 %	0.4 – 1.5 %

Can these conditions be met for all cases?

$10 \leq A \leq 38$ ←

$42 \leq A \leq 54$

$A \geq 62$

ISOSPIN SYMMETRY BREAKING CORRECTIONS

C = C1 + C2

C1	C2
Difference in configuration mixing between parent and daughter.	Mismatch in radial wave function between parent and daughter.
? • Shell-model calculation with well-established two-body matrix elements.	✓ • Full-parentage Wood-Saxon wave function matched to known binding energy and charge radius from electron scattering.
✓ • Charge dependence tuned to known single-particle energies and to measured IMME coefficients.	✓ • Compared with Hartree-Fock calculation matched to known binding energy.
✓ • Results also adjusted to measured 0^+ state energies.	✓ • Core states included based on measured spectroscopic factors.
0.01 – 0.3 %	0.4 – 1.5 %

Can these conditions be met for all cases?

$10 \leq A \leq 38$

$42 \leq A \leq 54$ ←

$A \geq 62$

ISOSPIN SYMMETRY BREAKING CORRECTIONS

$$C = C1 + C2$$

C1

Difference in configuration mixing between parent and daughter.

- ✘ Shell-model calculation with well-established two-body matrix elements.
- ✘ Charge dependence tuned to known single-particle energies and to measured IMME coefficients.
- ✓ Results also adjusted to measured 0^+ state energies.

0.01 – 0.3 %

C2

Mismatch in radial wave function between parent and daughter.

- ? Full-parentage Wood-Saxon wave function matched to known binding energy and charge radius from electron scattering.
- ✓ Compared with Hartree-Fock calculation matched to known binding energy.
- ✘ Core states included based on measured spectroscopic factors.

0.4 – 1.5 %

Can these conditions be met for all cases?

$10 \leq A \leq 38$

$42 \leq A \leq 54$

$A \geq 62$ ←

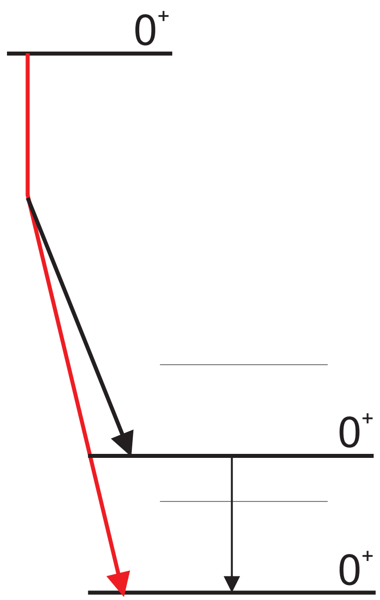
ISOSPIN SYMMETRY BREAKING CORRECTIONS

C = C1 + C2

Difference in configuration mixing between parent and daughter.

Mismatch in radial wave function between parent and daughter.

Experimental control for $_{C1}$ ✔



$\langle M_1 \rangle^2 = 2 \,'_{C1}$

$\langle M_0 \rangle^2 = 2(1- {}_{C1})$

Branching ratios to non-analogue 0^+ states

Parent	Experimental results (ppm)	Calculation (ppm)
^{38m}K	< 12	6 ± 2
^{42}Sc	59 ± 14	22 ± 22
^{46}V	39 ± 4	18 ± 14
^{50}Mn	< 3	8 ± 4
^{54}Co	45 ± 6	65 ± 25
^{62}Ga	53 ± 25	240 ± 80

ISOSPIN SYMMETRY BREAKING CORRECTIONS

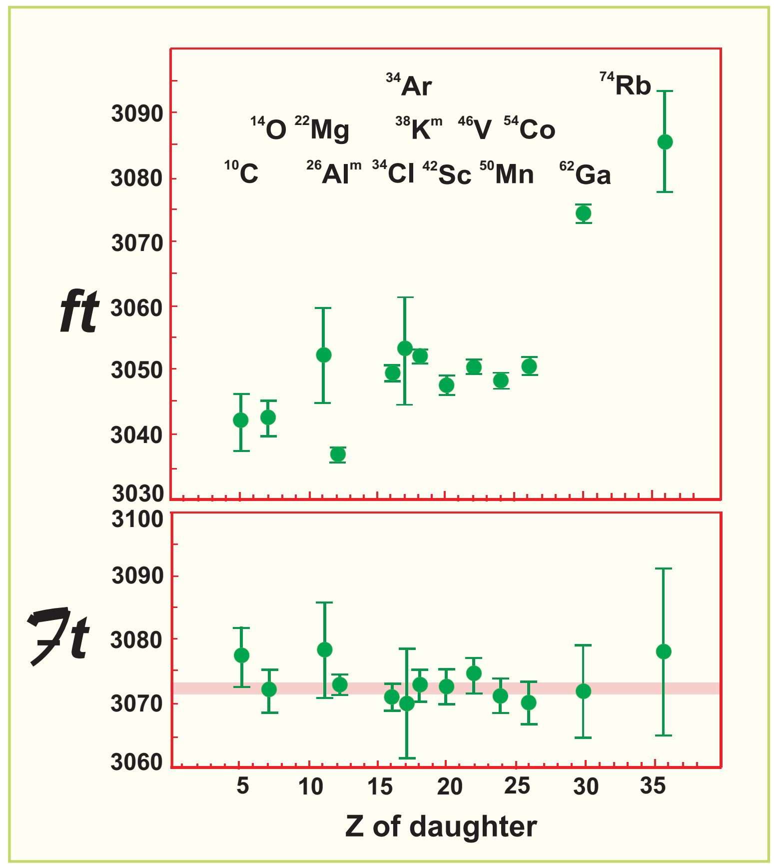
C = C1 + C2

Difference in configuration mixing between parent and daughter.

Mismatch in radial wave function between parent and daughter.

Experimental control for C1 ✔

Experimental control for C ✔



ISOSPIN SYMMETRY BREAKING CORRECTIONS

$_{C}$ = $_{C1}$ + $_{C2}$

Difference in configuration mixing between parent and daughter.

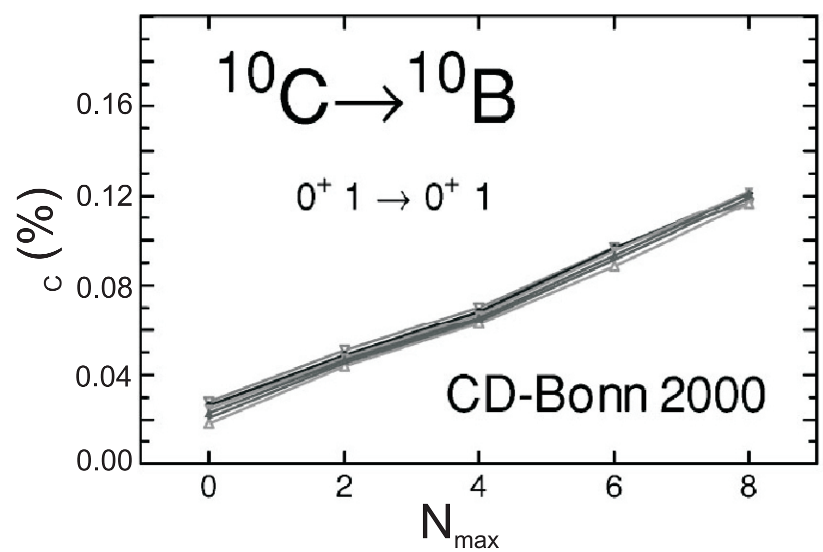
Mismatch in radial wave function between parent and daughter.

Experimental control for $_{C1}$ ✔

Experimental control for $_{C}$ ✔

Theoretical control for $_{C}$ ✔

Ab initio shell model calculation up to 8ħ
Caurier *et al*., PRC 66, 024314 (2002)]



No convergence for $_{C}$ with N up to N_{max} = 8

Full $_{C}$ estimated by perturbation theory: 0.19%

Our result: $_{C}$ = 0.18(2)%

TESTING $_{C}$ CALCULATIONS AGAINST CVC EXPECTATIONS

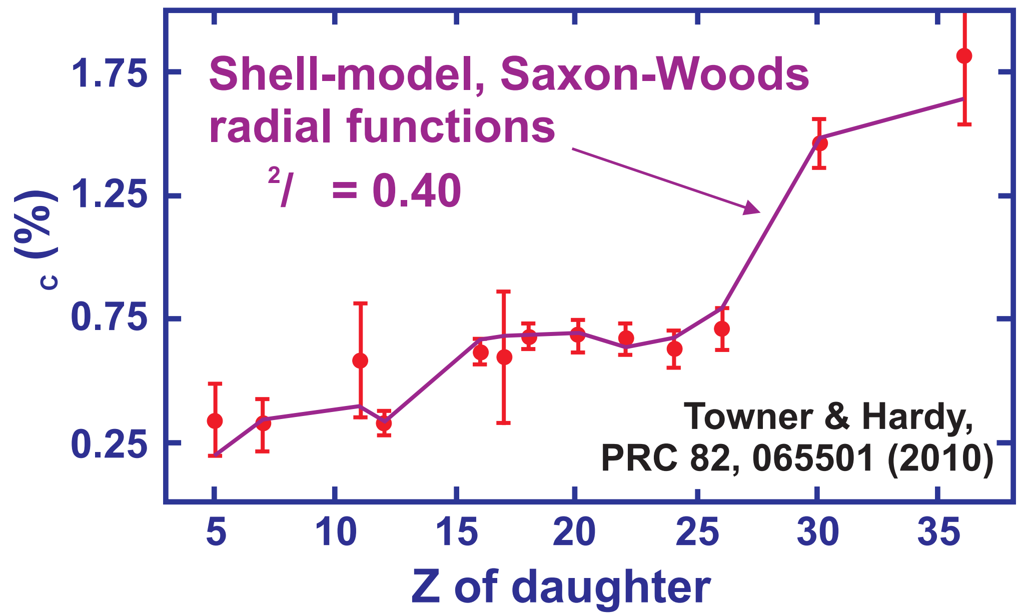
$$\mathcal{F}t = ft\,(1 + {}'_{R})[1 - ({}_{C} - {}_{NS})] = \frac{K}{2G_V^{\,2}\,(1 + {}_{R})}$$

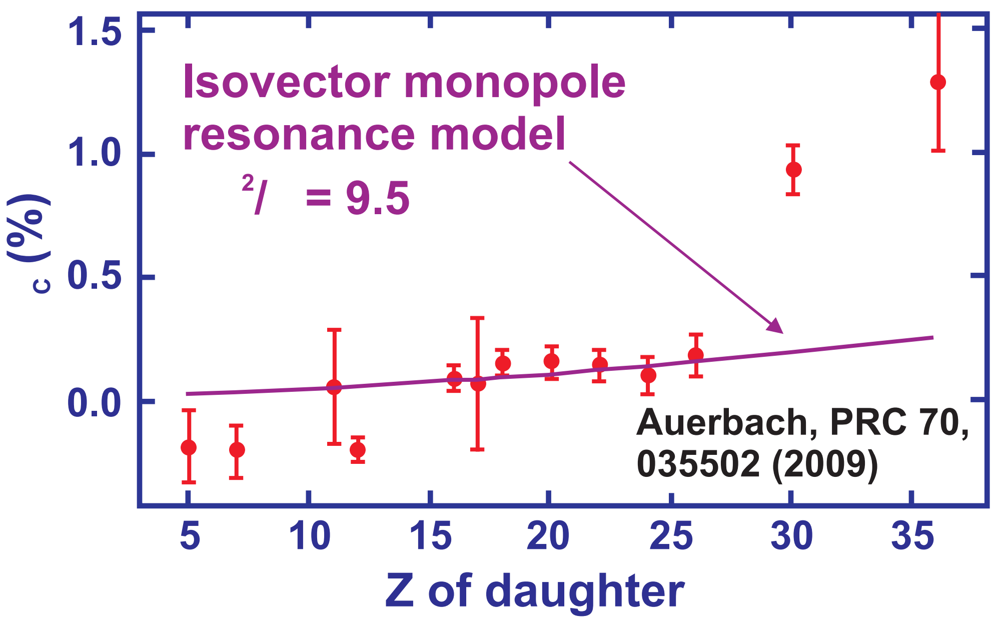
To satisfy CVC,

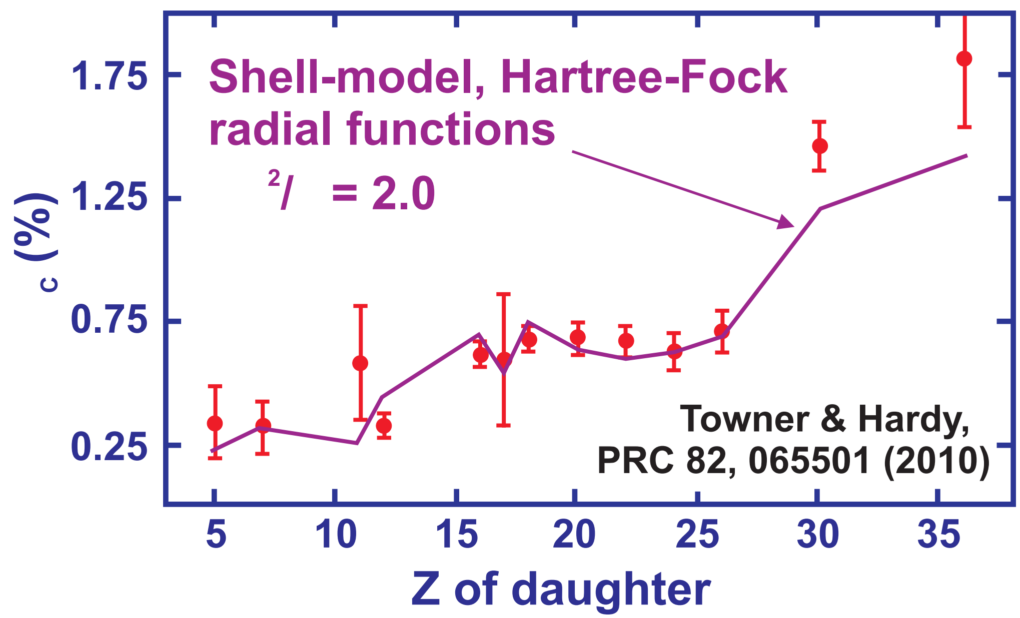
$$ft\,(1 + {}'_{R})[1 - ({}_{C} - {}_{NS})] = A$$

where A takes the same value for all measured transitions. Therefore

$${}_{C} = 1 + {}_{NS} - \frac{A}{ft\,(1 + {}'_{R})}$$







TESTING δ_C CALCULATIONS

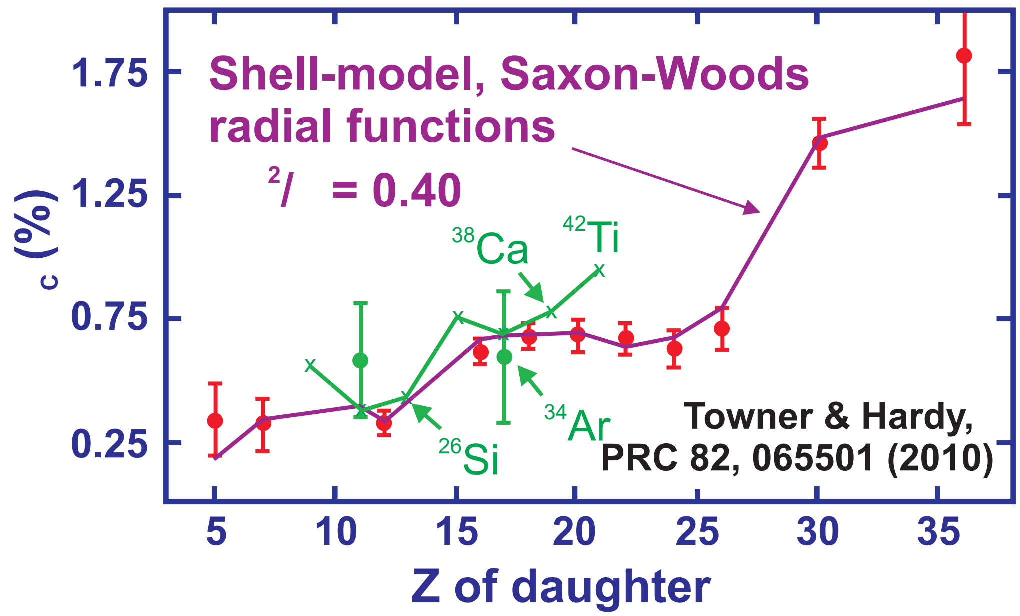
$$\mathcal{F}t = ft\,(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

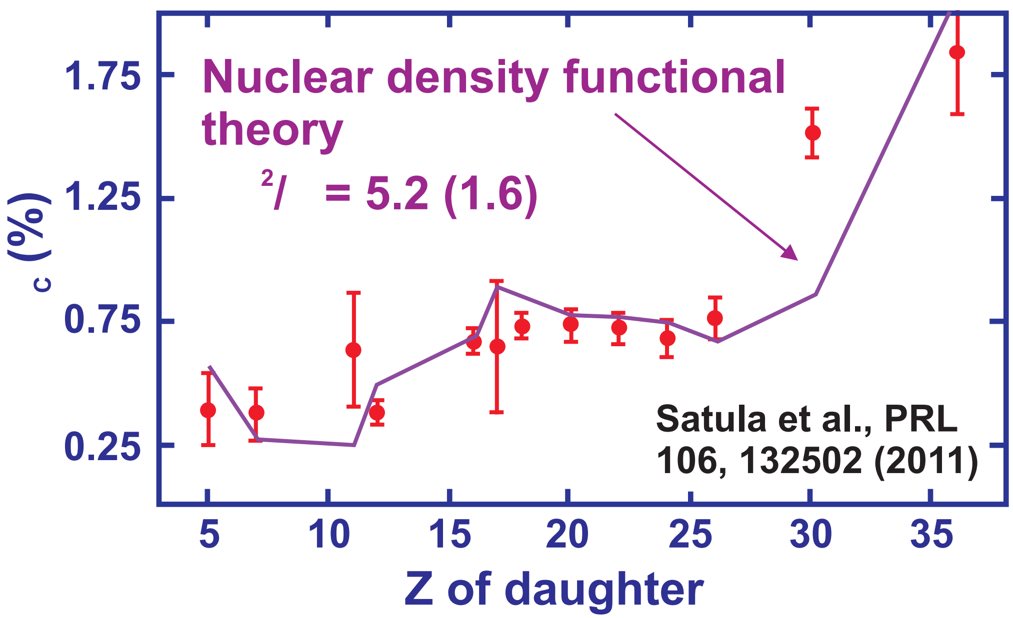
To satisfy CVC,

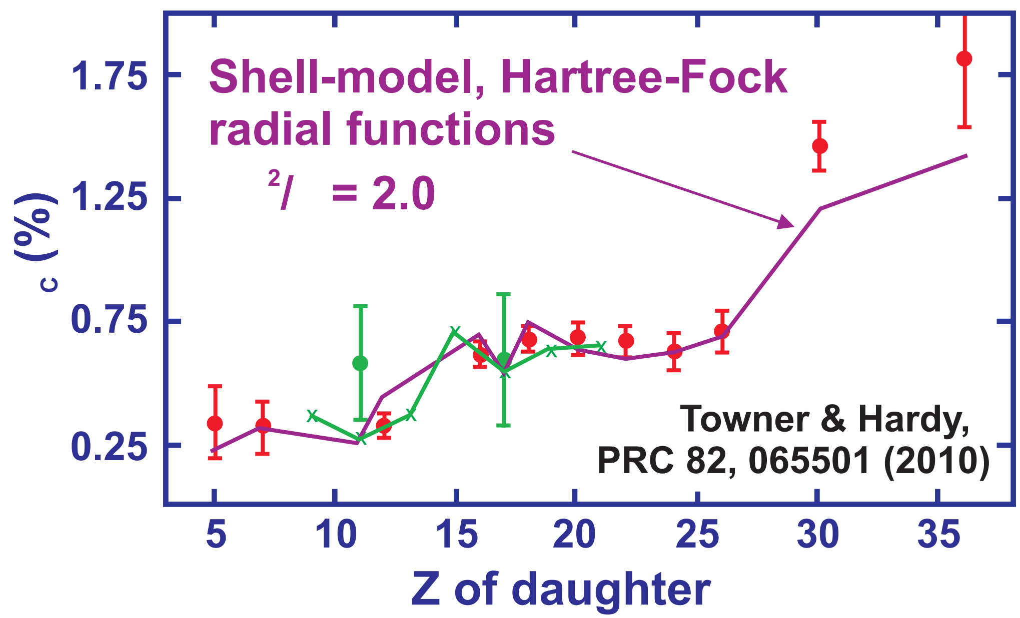
$$ft\,(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = A$$

where A takes the same value for all measured transitions. Therefore

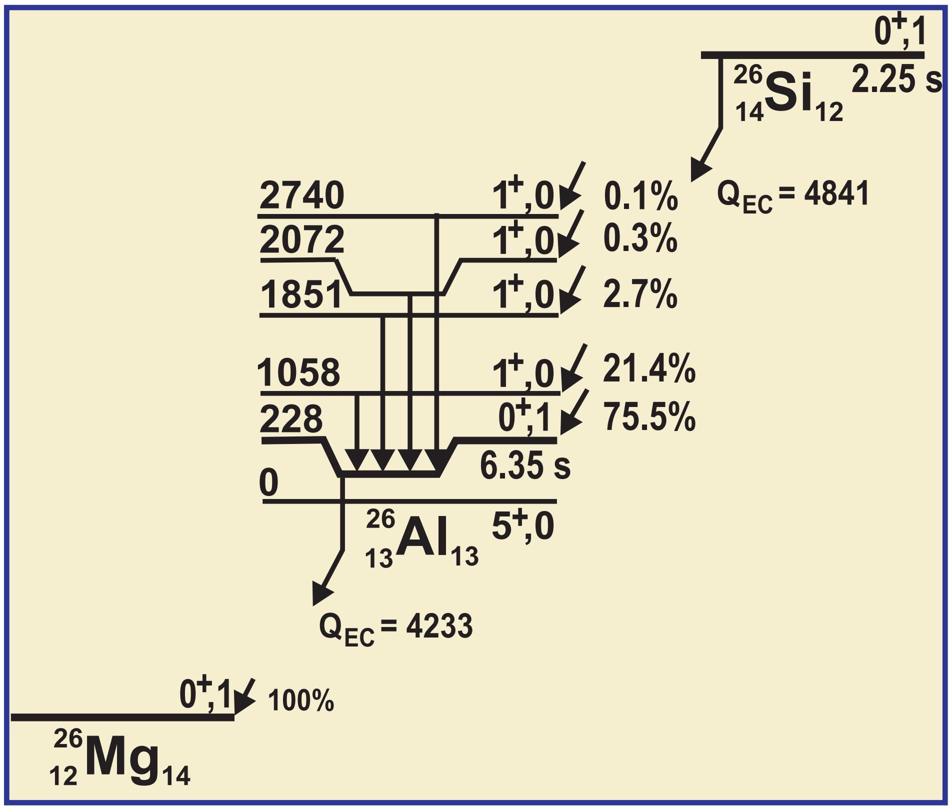
$$\delta_C = 1 + \delta_{NS} - \frac{A}{ft\,(1 + \delta'_R)}$$





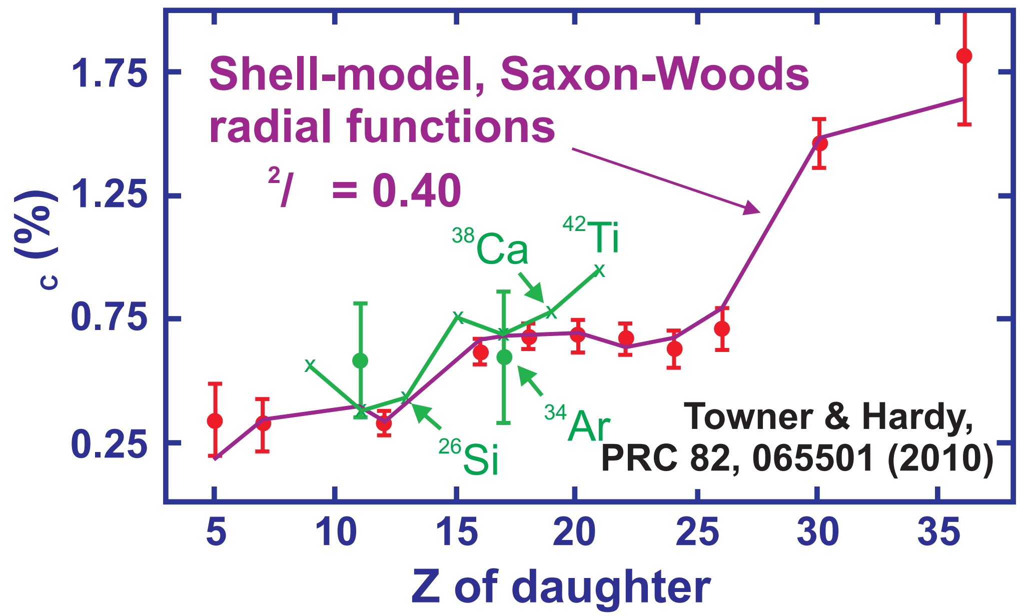


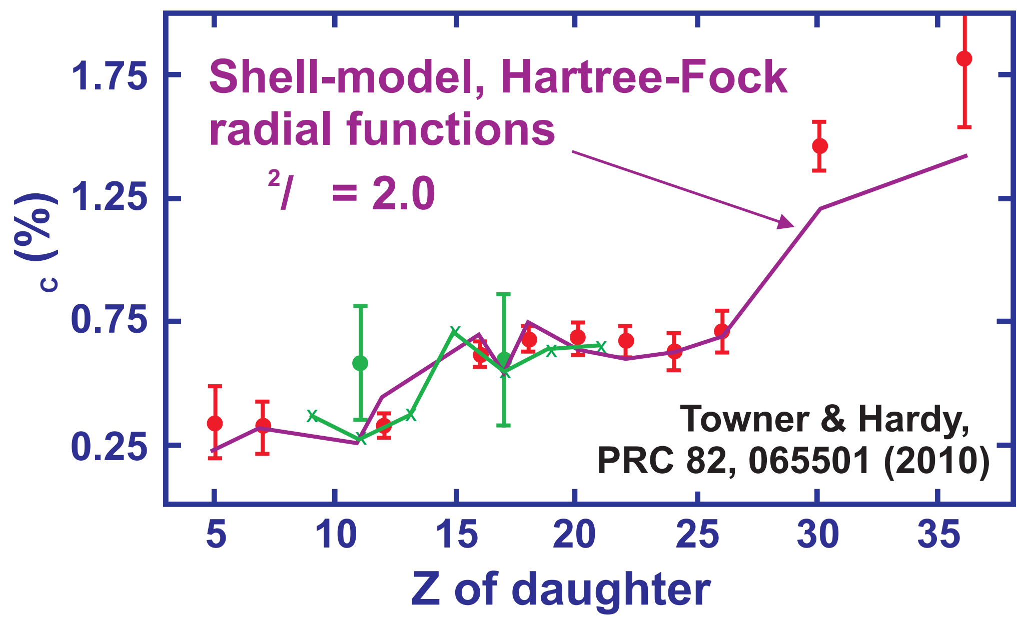
TESTING $_C$ CALCULATIONS



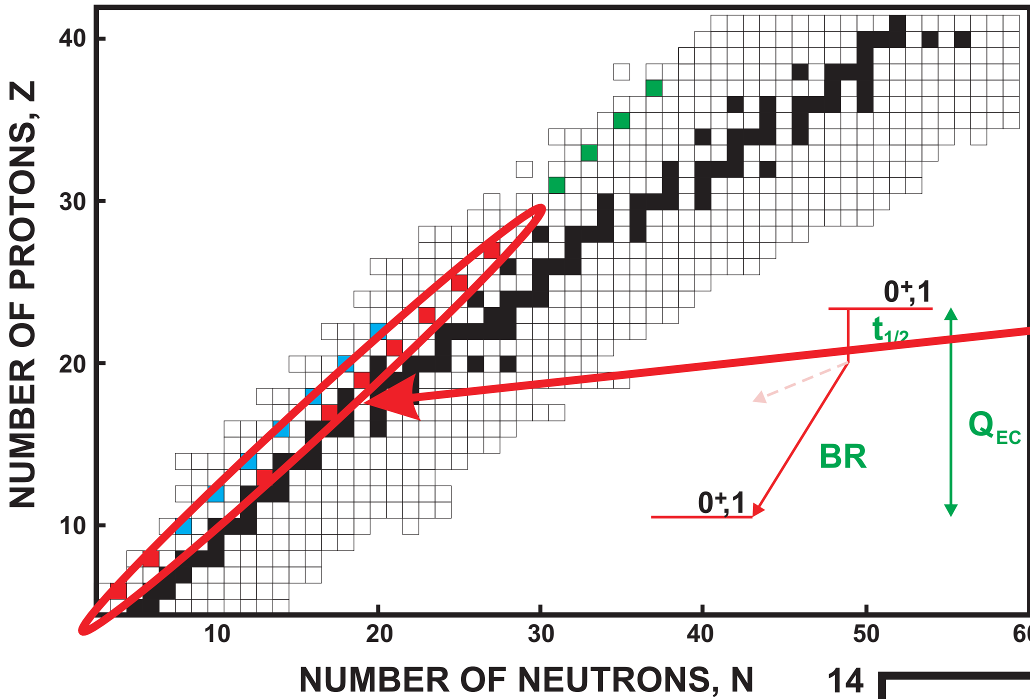
T_z = 0 decay

T_z = -1 decay

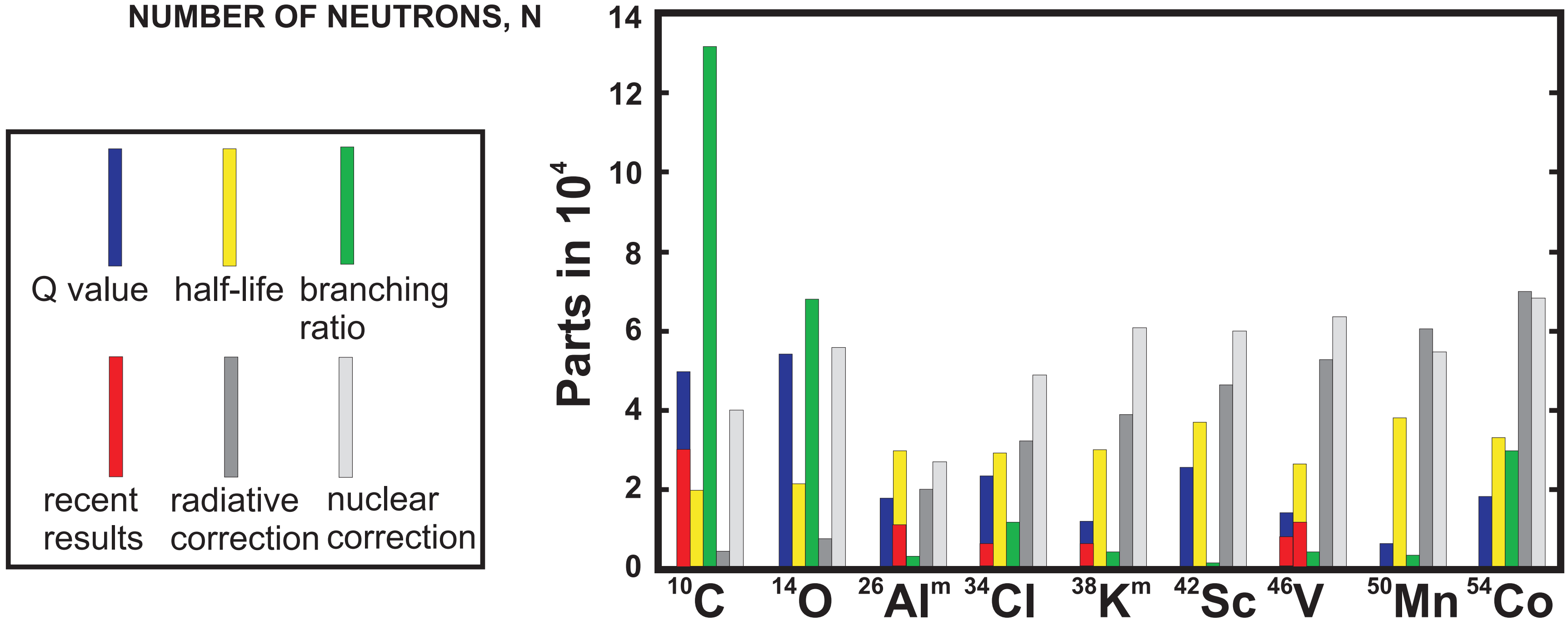




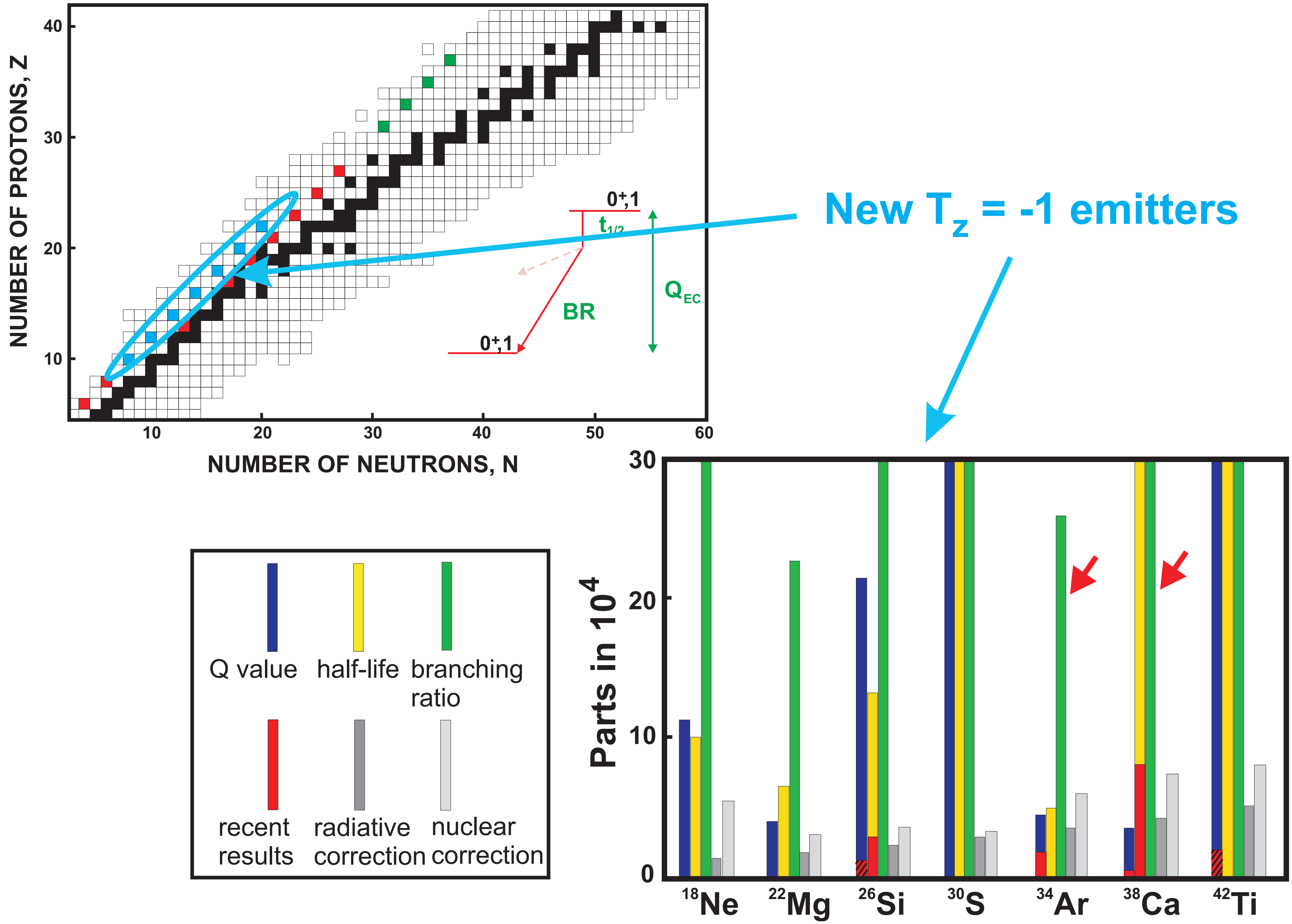
IMPROVEMENTS SINCE 2009



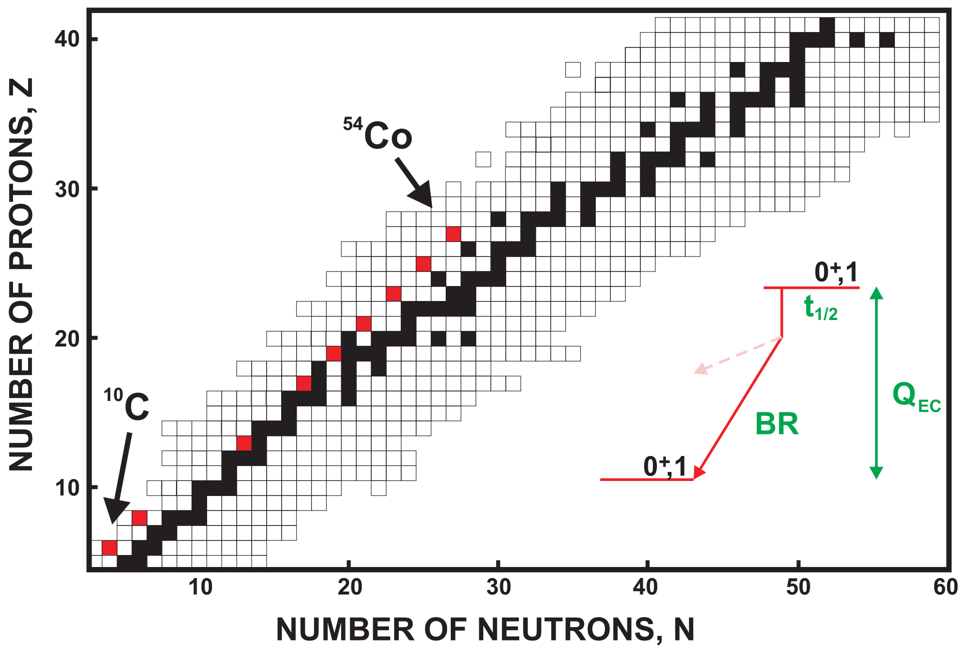
Traditional 9 emitters



IMPROVEMENTS SINCE 2009



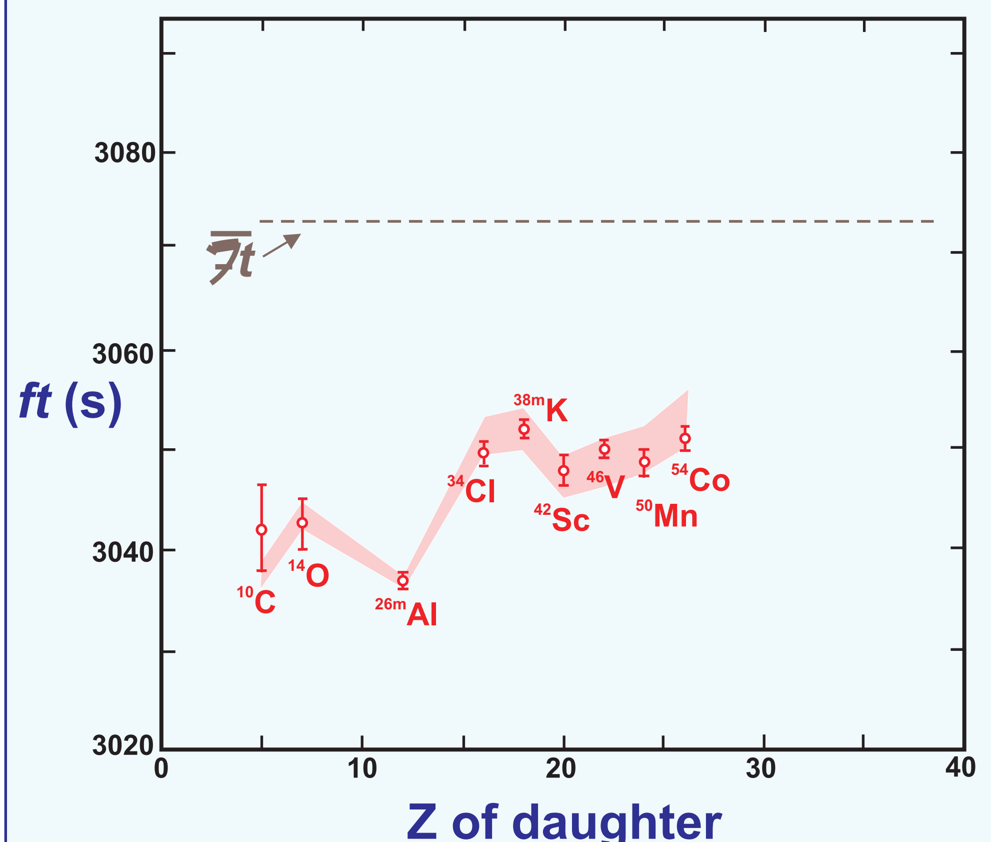
CURRENT DIRECTION OF NUCLEAR EXPERIMENTS



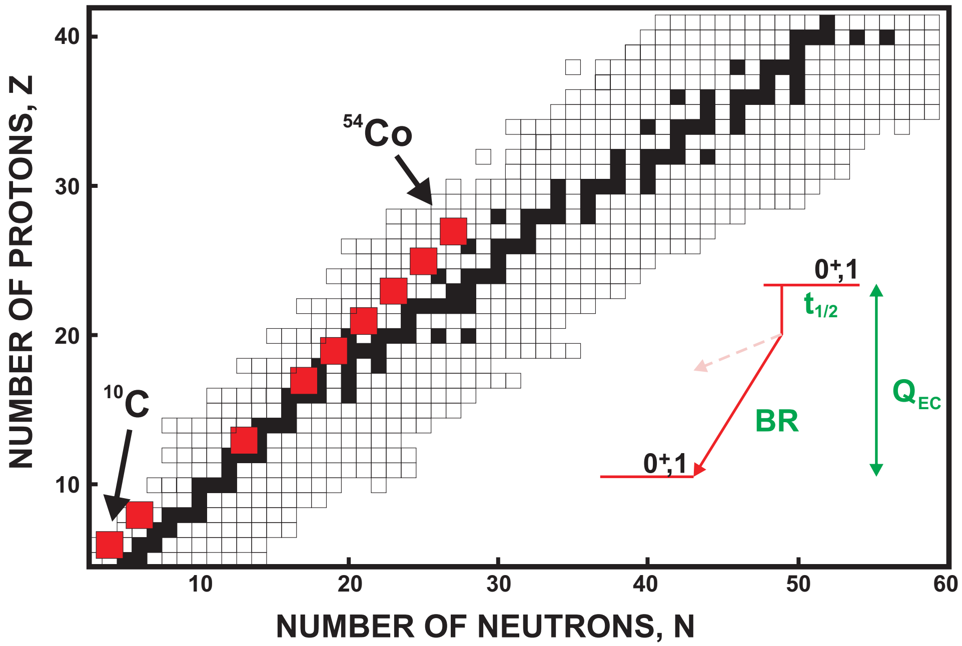
$$\mathcal{F}t = ft\,(1 + {}'_R)[1 - ({}_C - {}_{NS})] = \frac{K}{2G_V^2\,(1 + {}_R)}$$

Strategy is to probe the nucleus-to-nucleus variation in $_C$ - $_{NS}$.

$$\text{Calculated } ft\text{-value} = \frac{\overline{\mathcal{F}t}}{(1 + {}'_R)[1 - ({}_C - {}_{NS})]}$$



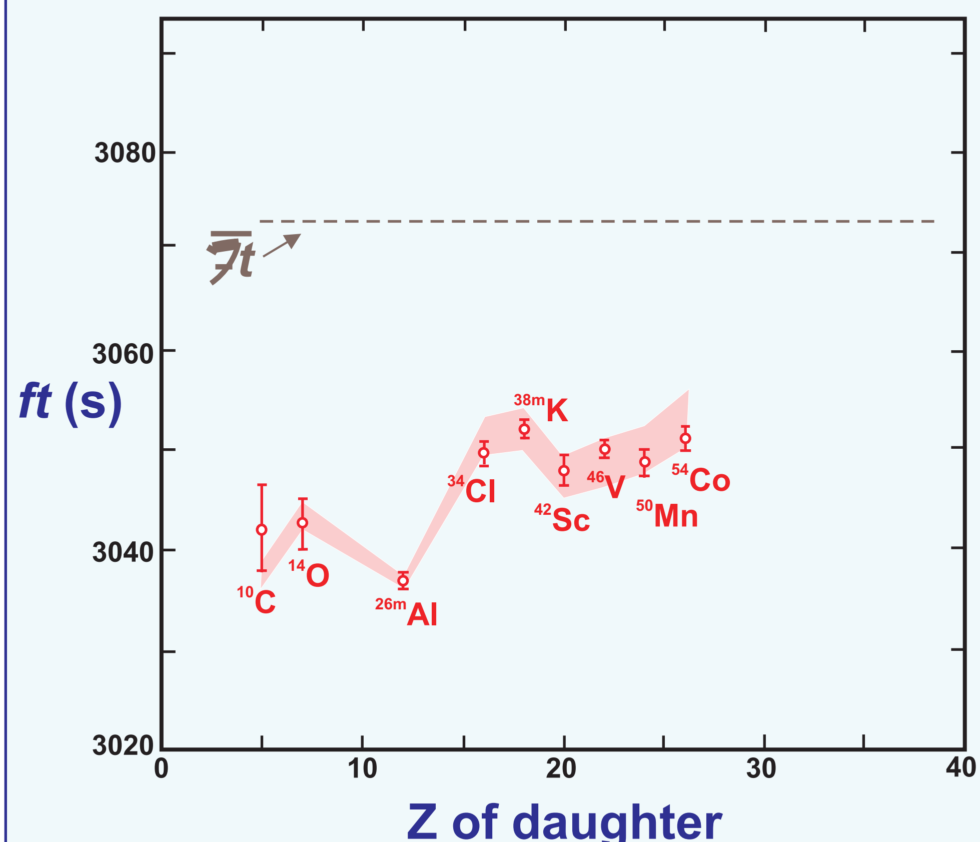
CURRENT DIRECTION OF NUCLEAR EXPERIMENTS



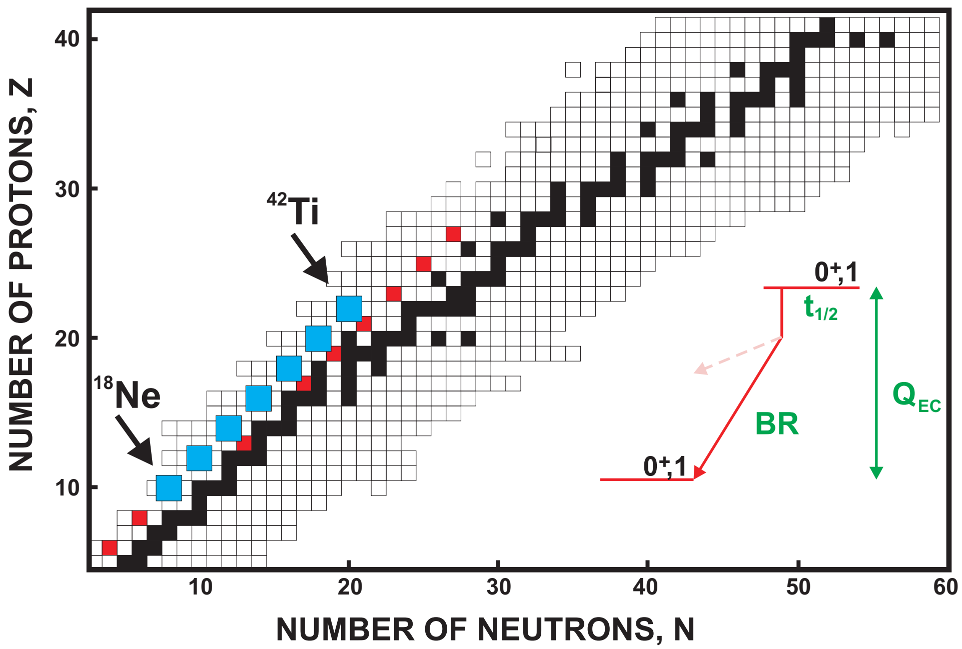
Strategy is to probe the nucleus-to-nucleus variation in $_{C}$ - $_{NS}$.

* Increase measured precision on nine best *ft*-values

$$\text{Calculated } ft\text{-value} = \frac{\overline{\mathcal{F}t}}{(1 + \ '_{R})[1 - (\ _{C} - \ _{NS})]}$$



CURRENT DIRECTION OF NUCLEAR EXPERIMENTS

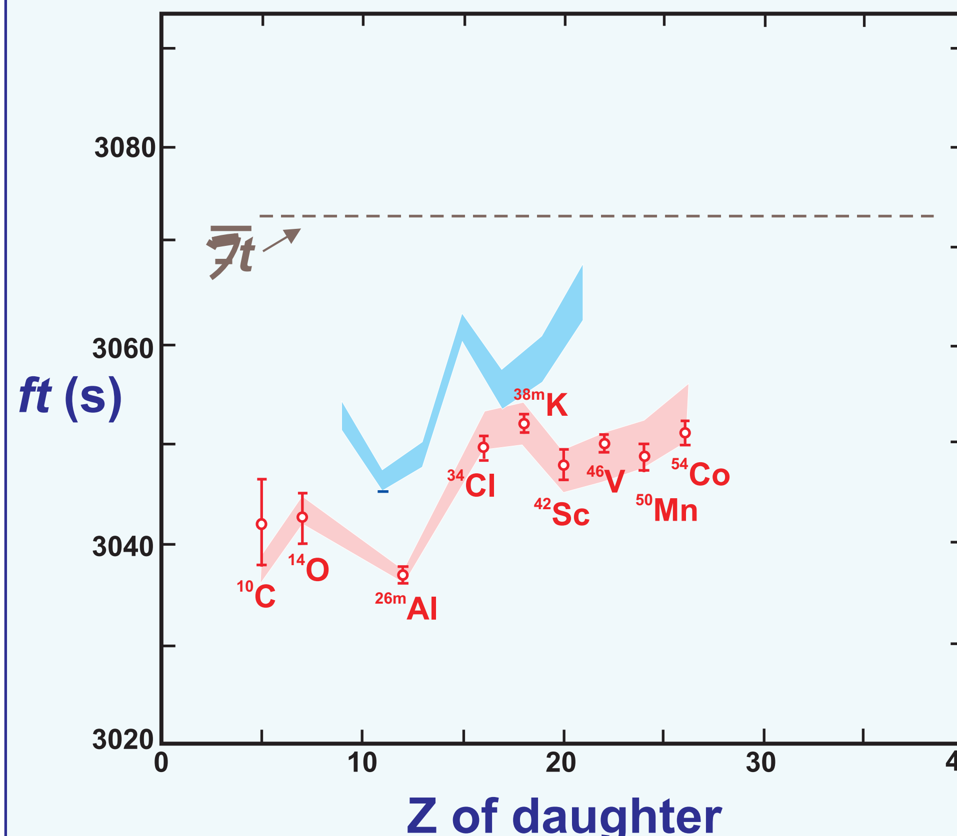


* Increase measured precision on nine best *ft*-values

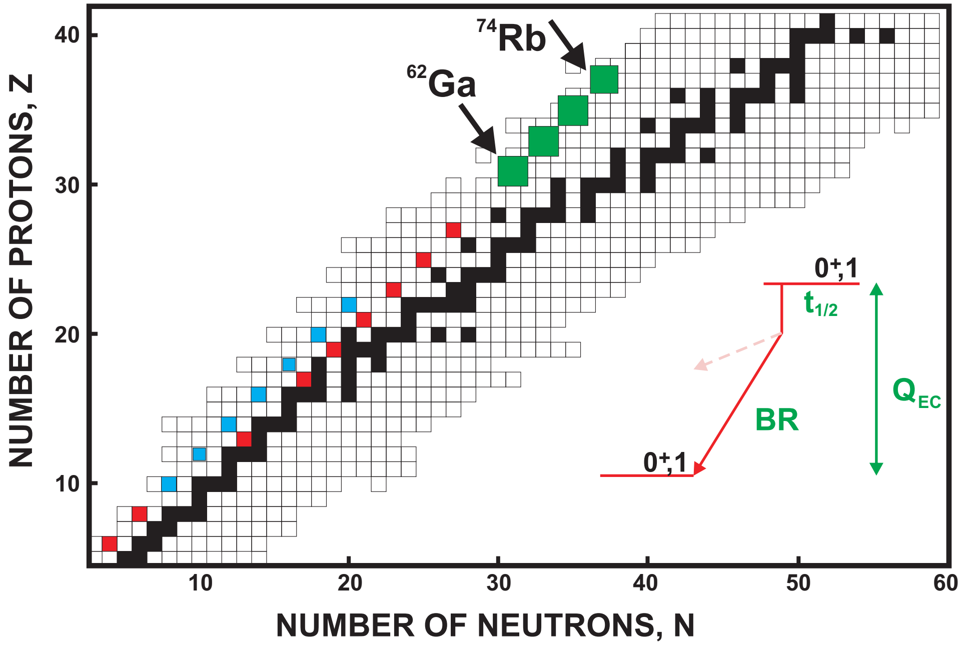
* measure new $0^+ \rightarrow 0^+$ decays with $18 \leq A \leq 42$ (T_Z = -1)

Strategy is to probe the nucleus-to-nucleus variation in $_C$ - $_{NS}$.

Calculated *ft*-value = $\dfrac{\mathcal{F}t}{(1 + \ '_R)[1 - (\ _C - \ _{NS})]}$

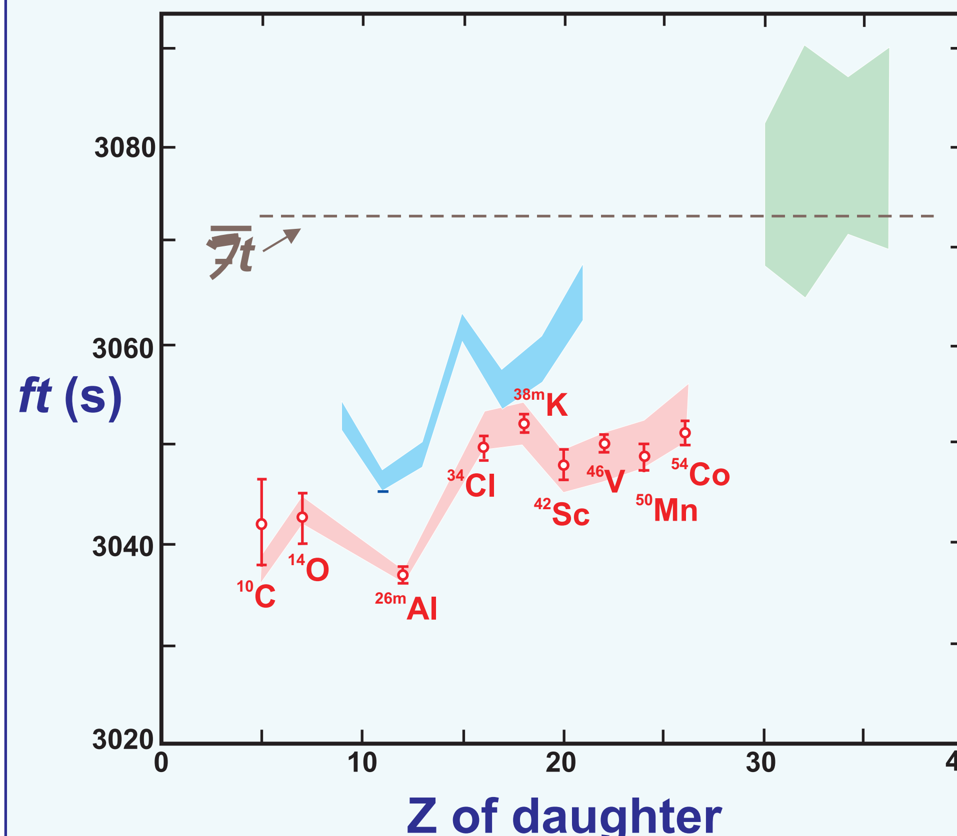


CURRENT DIRECTION OF NUCLEAR EXPERIMENTS



Strategy is to probe the nucleus-to-nucleus variation in $_{C}$ - $_{NS}$.

Calculated *ft*-value = $\dfrac{\overline{\mathcal{F}t}}{(1 + {}'_{R})[1 - ({}_{C} - {}_{NS})]}$

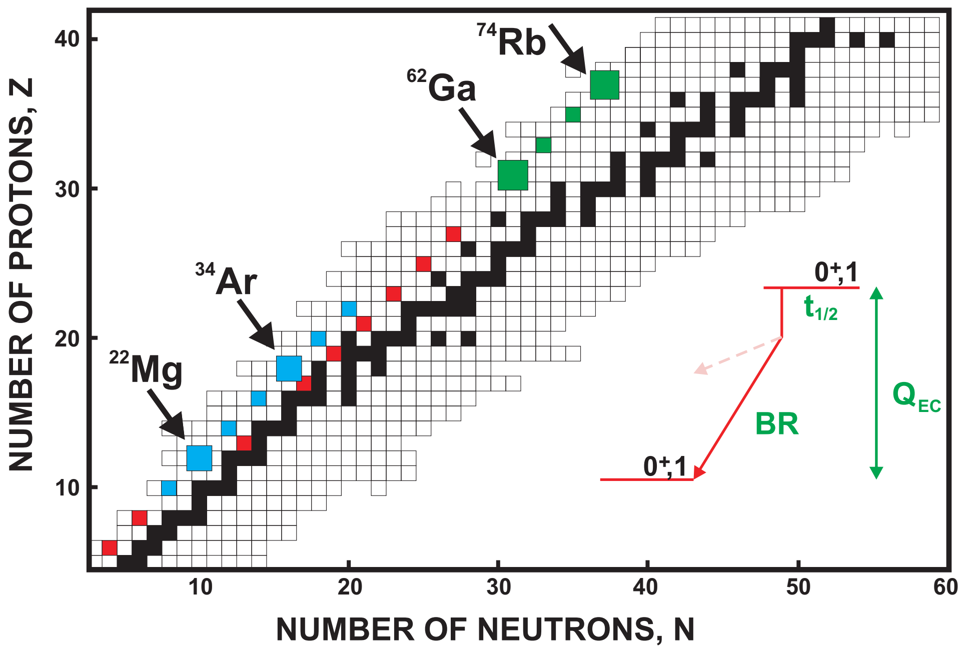


* **Increase measured precision on nine best *ft*-values**

* **measure new $0^+ \rightarrow 0^+$ decays with $18 \leq A \leq 42$ (T_Z = -1)**

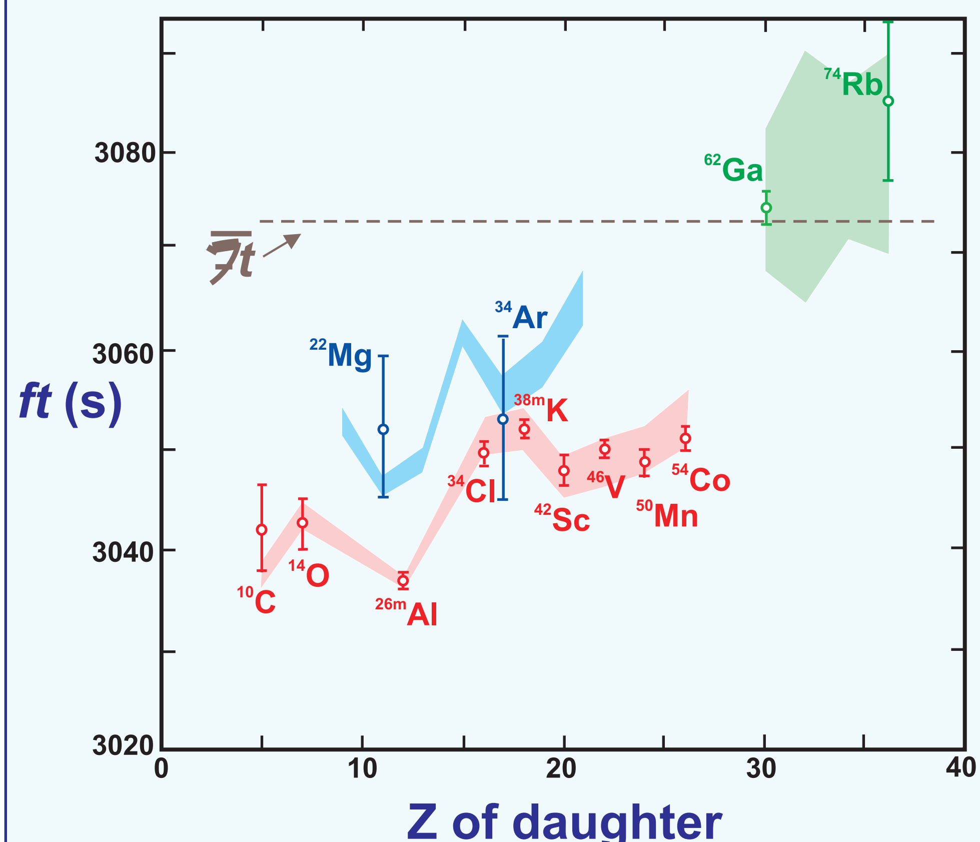
* **measure new $0^+ \rightarrow 0^+$ decays with $A \geq 62$ (T_Z = 0)**

CURRENT DIRECTION OF NUCLEAR EXPERIMENTS



Strategy is to probe the nucleus-to-nucleus variation in $_{C}$ - $_{NS}$.

Calculated *ft*-value = $\dfrac{\overline{\mathcal{F}t}}{(1 + {}'_{R})[1 - ({}_{C} - {}_{NS})]}$



* **Increase measured precision on nine best *ft*-values**

* **measure new $0^+ \rightarrow 0^+$ decays with $18 \leq A \leq 42$ (T_z = -1)**

* **measure new $0^+ \rightarrow 0^+$ decays with $A \geq 62$ (T_z = 0)**

SUMMARY AND OUTLOOK

1. Analysis of superallowed $0^+ \rightarrow 0^+$ nuclear decay is shown to confirm CVC and thus yield V_{ud} = 0.97425(22).
2. The three other experimental methods for determining V_{ud} yield consistent results, but are less precise by a factor of 8 or more.
3. The current value for V_{ud}, when combined with V_{us} and V_{ub}, satisfies CKM unitarity to 0.06%.

4. The largest contribution to the V_{ud} uncertainty is from the inner radiative correction. Isospin symmetry-breaking corrections in nuclei are the second largest.
5. These symmetry-breaking corrections can be tested by requiring consistency among 13 known transitions (CVC). Standard corrections pass the test; a few others do too.
6. They can be further tested and improved by higher experimental precision and by new transitions from T_z = -1 parents.