

# SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

## BASIC WEAK-DECAY EQUATION

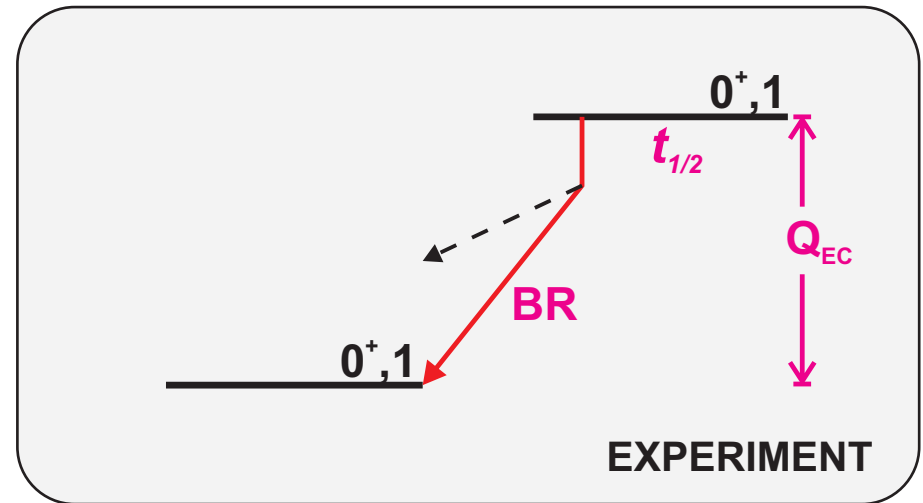
$$ft = \frac{K}{G_V^2 \langle \rangle^2}$$

$f$  = statistical rate function:  $f(Z, Q_{EC})$

$t$  = partial half-life:  $f(t_{1/2}, BR)$

$G_V$  = vector coupling constant

$\langle \rangle$  = Fermi matrix element



Reference: Hardy & Towner,  
PRC 79, 055502 (2009)

# SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

## BASIC WEAK-DECAY EQUATION

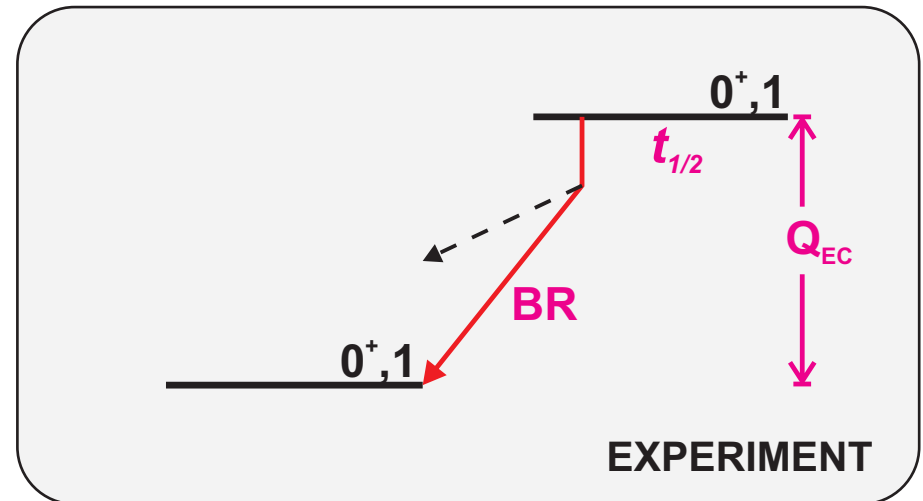
$$ft = \frac{K}{G_V^2 \langle \rangle^2}$$

$f$  = statistical rate function:  $f(Z, Q_{EC})$

$t$  = partial half-life:  $f(t_{1/2}, BR)$

$G_V$  = vector coupling constant

$\langle \rangle$  = Fermi matrix element



## INCLUDING RADIATIVE AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS

$$\mathcal{F}t = ft (1 + \overset{R}{\prime}) [1 - \overset{C}{\text{---}} \overset{NS}{\text{---}}] = \frac{K}{2G_V^2 (1 + \overset{R}{\prime})}$$

$f(Z, Q_{EC})$

~1.5%

$f(\text{nuclear structure})$

0.3-1.5%

$f(\text{interaction})$

~2.4%

THEORETICAL UNCERTAINTIES

0.05 – 0.10%

# WHAT CAN WE LEARN?

## FROM A SINGLE TRANSITION

Experimentally  
determine  $G_V^2(1 + \epsilon_R)$

$$\tau_t = \tau_t (1 + \epsilon_R) [1 - (\epsilon_C - \epsilon_{NS})] = \frac{K}{2G_V^2(1 + \epsilon_R)}$$

## FROM MANY TRANSITIONS

Test Conservation of  
the Vector current (CVC)

Validate the correction  
terms

Test for presence of  
a Scalar current

$\tau_t$  values constant

## WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates      Cabibbo Kobayashi Maskawa (CKM) matrix      mass eigenstates

Obtain precise value of  $G_V^2(1 + \epsilon_R)$   
Determine  $V_{ud}^2$

$$V_{ud}^2 = G_V^2 / G^2$$

Test CKM unitarity

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

# WHAT CAN WE LEARN?

## FROM A SINGLE TRANSITION

Experimentally determine  $G_V^2 (1 + R)$

$$\tau_t = \tau_t (1 + R) [1 - (C - NS)] = \frac{K}{2G_V^2 (1 + R)}$$

## FROM MANY TRANSITIONS

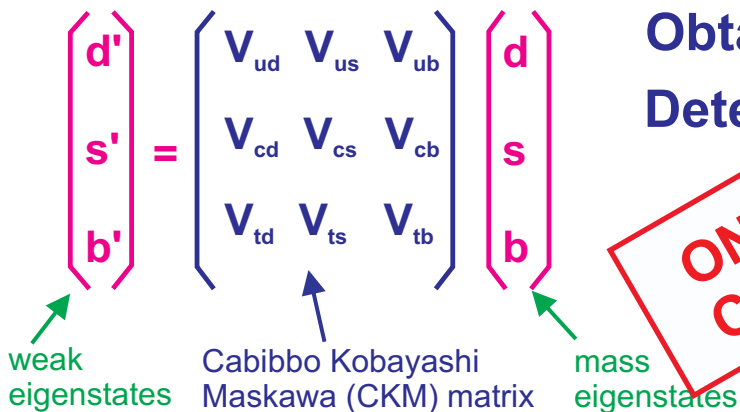
Test Conservation of the Vector current (CVC)

Validate the correction terms

Test for presence of a Scalar current

$$\tau_t \text{ values constant}$$

## WITH CVC VERIFIED



Obtain precise  $G_V^2 (1 + R)$   
Determine  $\tau_t$

**ONLY POSSIBLE IF PRIOR CONDITIONS SATISFIED**

unitarity

$$G_V^2 (1 + R)$$

$$V_{ud}^2 = G_V^2 / G^2$$

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

# CKM MATRIX AND UNITARITY

CABIBBO-KOBAYASHI-MASKAWA  
QUARK-MIXING MATRIX

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates      mass eigenstates

THREE-GENERATION  
UNITARITY

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

Reference: Berennger *et al.* [PDG],  
PRD **86**, 010001 (2012)

# CKM MATRIX AND UNITARITY

CABIBBO-KOBAYASHI-MASKAWA  
QUARK-MIXING MATRIX

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates
mass eigenstates

This is the most demanding test available!

THREE-GENERATION  
UNITARITY

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

$V_{ud}^2 = G_V^2 / G^2$   
nuclear (n & ) decays  
muon decay

$$\begin{aligned}
 K^+ &\longrightarrow e^+ e^- \\
 K_L^0 &\longrightarrow e^\mp e^\mp
 \end{aligned}$$

**0.0507(4)**

B decays  
**0.000015**

WHAT PRECISION IS NEEDED?  $1 - V_{us}^2 - V_{ub}^2 = 0.9493(4)$   
**< 0.05%**

# PRECISION REQUIRED FROM EXPERIMENT

$$\tau t = ft (1 + \frac{R}{R'}) [1 - (C - NS)] = \frac{K}{2G_V^2 (1 + R)}$$

Precision required for CKM unitarity test: **< 0.05%**

Precision achievable for calculated corrections: **0.05-0.10%**

Required from experiment:

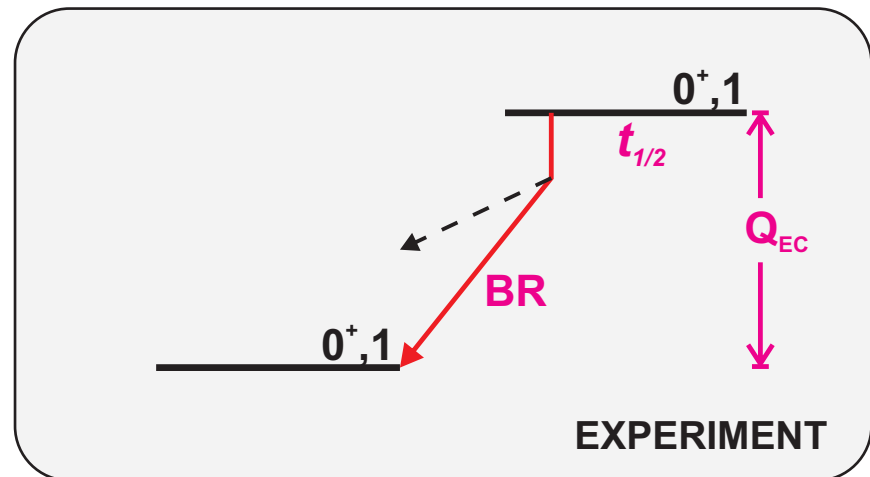
$$t = t_{1/2} / BR$$

Precision for  $t$  **0.05%**

$$f = f(Z, Q_{EC}) \propto Q^5$$

Precision for  $Q$  **0.01%**

200eV – 1keV



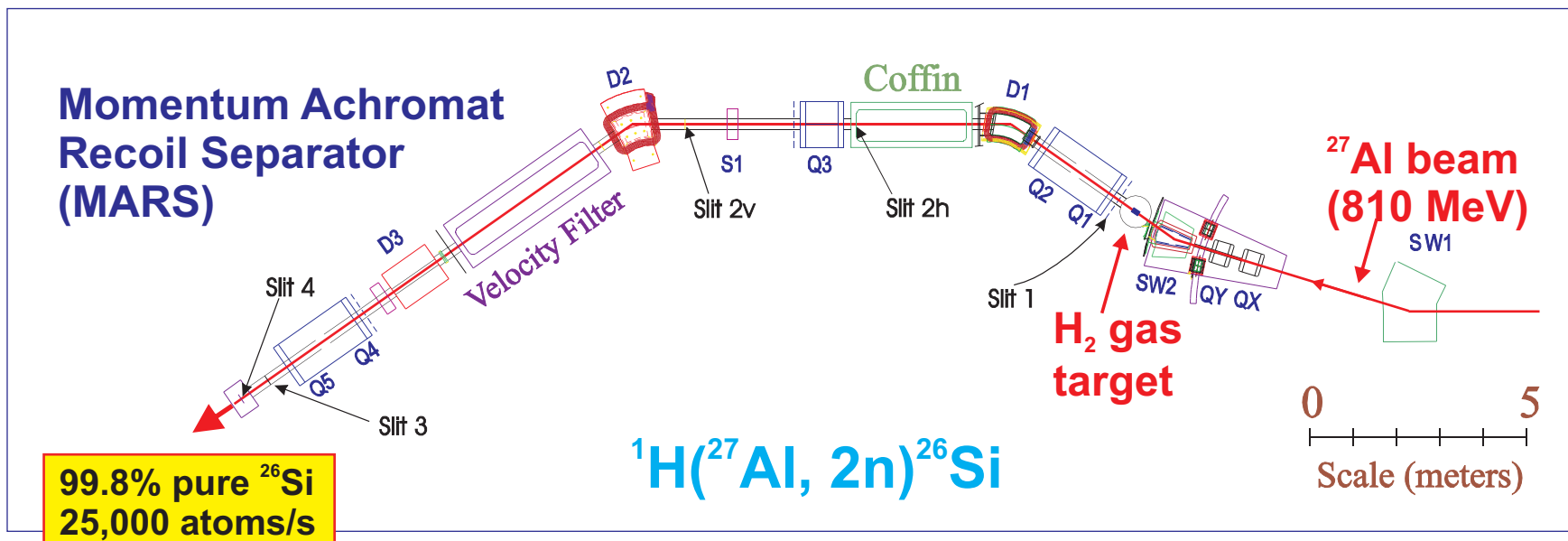
By the usual nuclear physics standards, these are very challenging requirements!

# GUIDELINES FOR PRECISION MEASUREMENTS

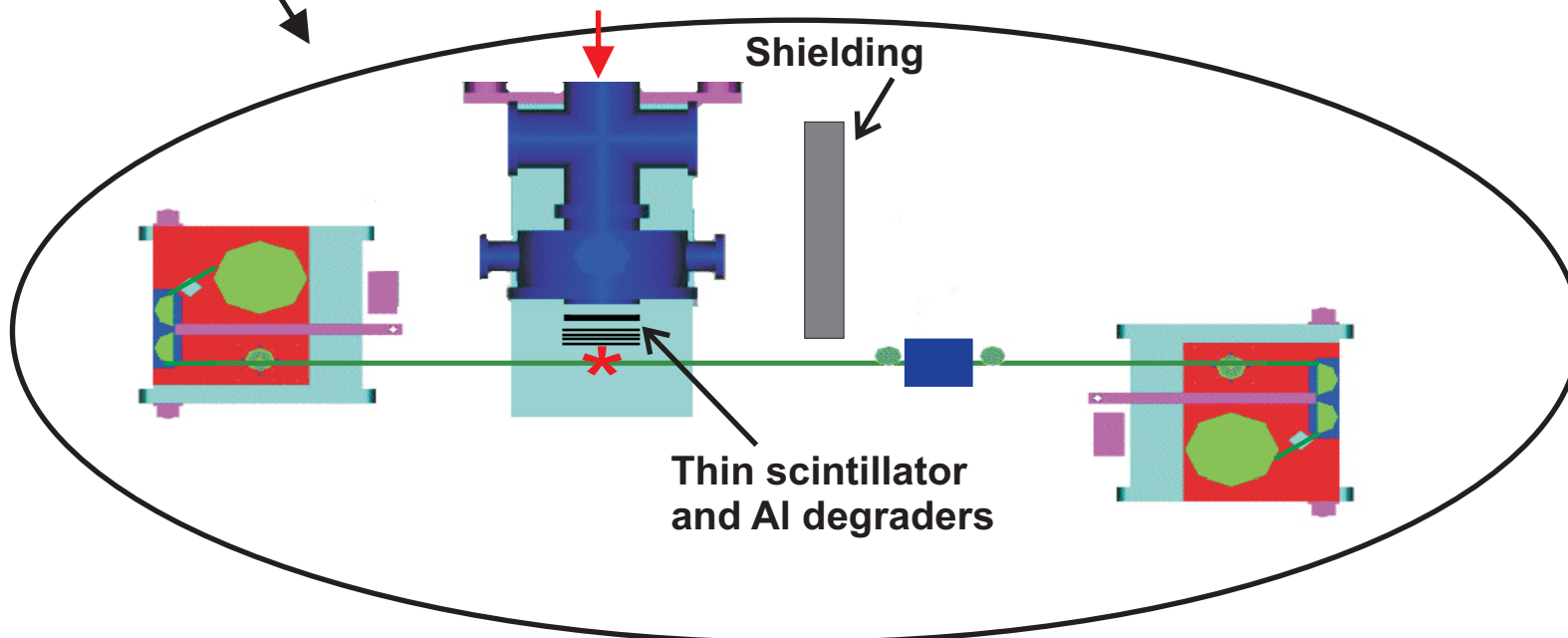
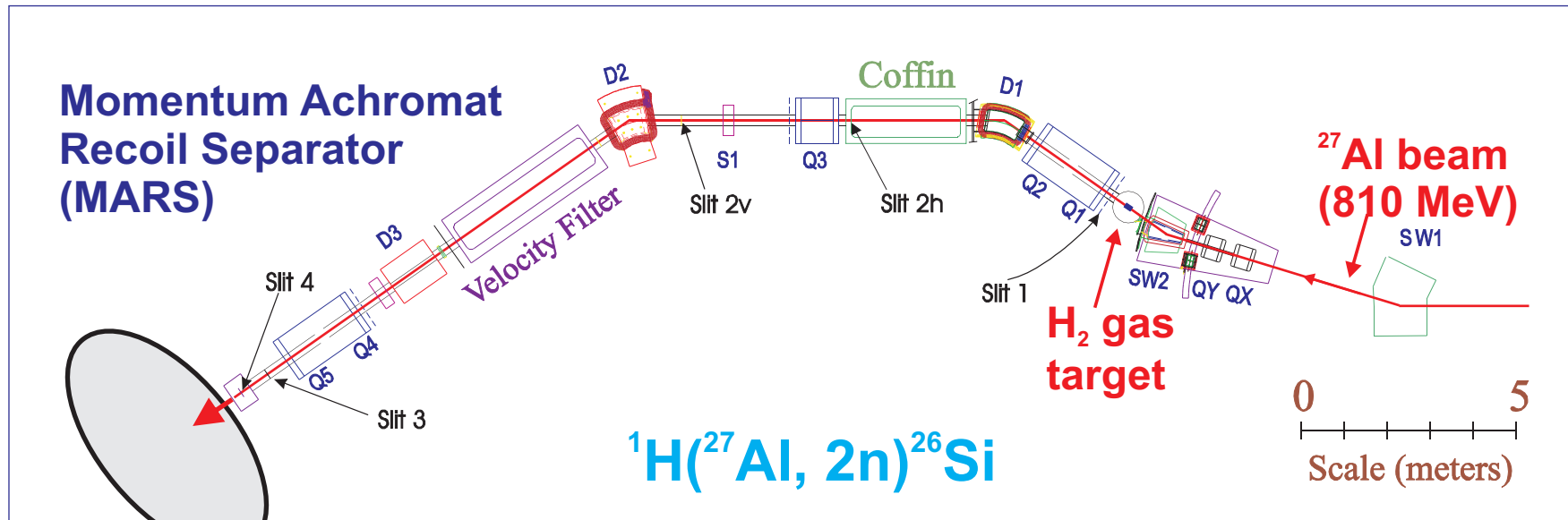
- **Experimental apparatus should be as simple as possible.**
- **All experimental parameters must be under control and testable.**
- **Experimental equipment should be dedicated only to this measurement.**
- **Calibration is often the most important part of the measurement.**
- **Tests for sources of systematic error must dominate data acquisition.**
- **Redundancy is desirable in both measurement and analysis.**
- **No inconsistencies can be overlooked.**
- **A complete error budget is the most important part of the result.**



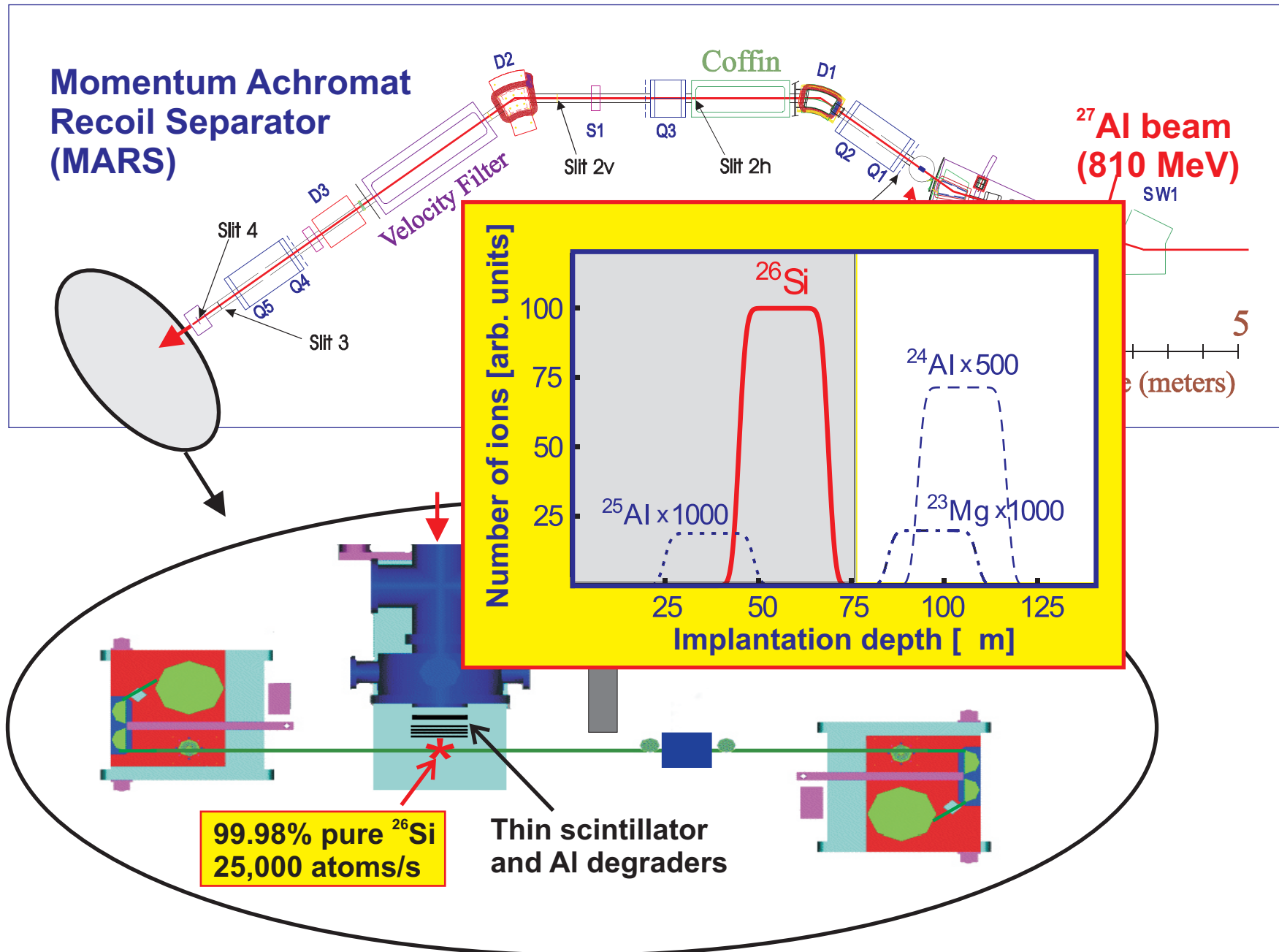
# PRECISION DECAY MEASUREMENTS AT TAMU



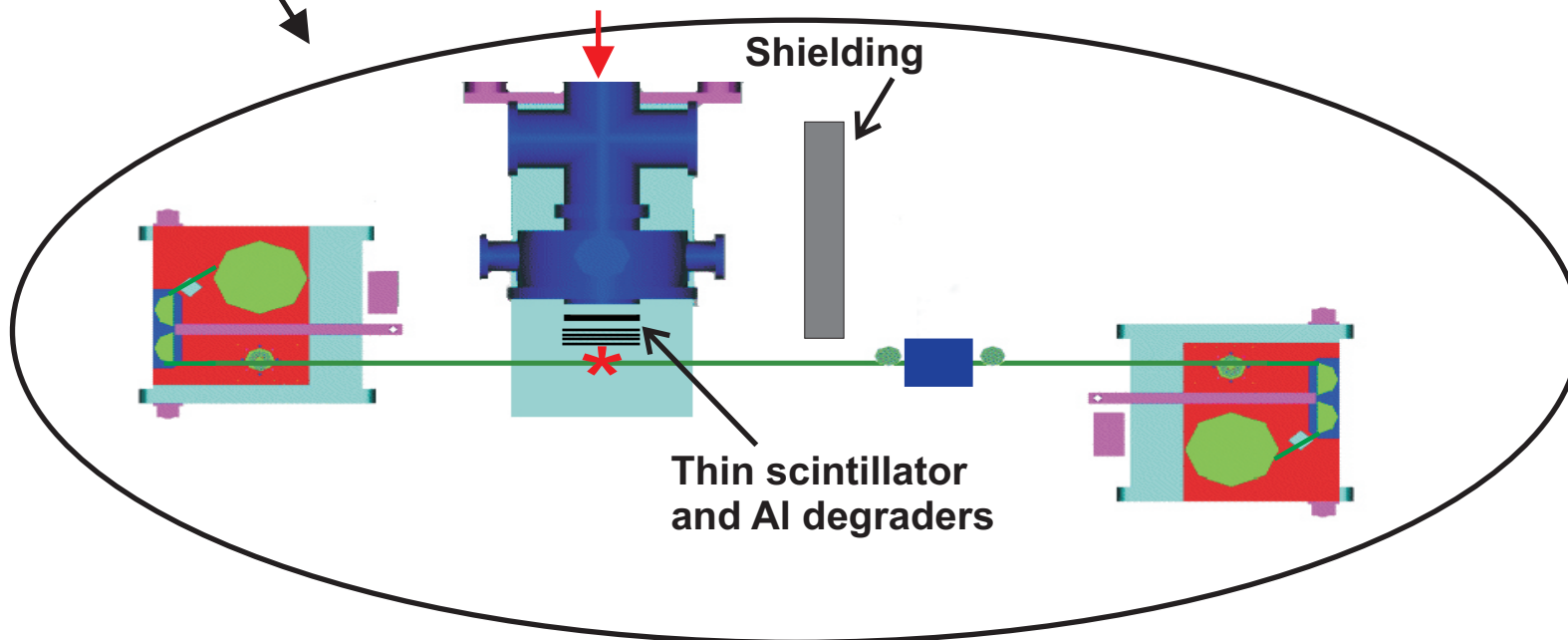
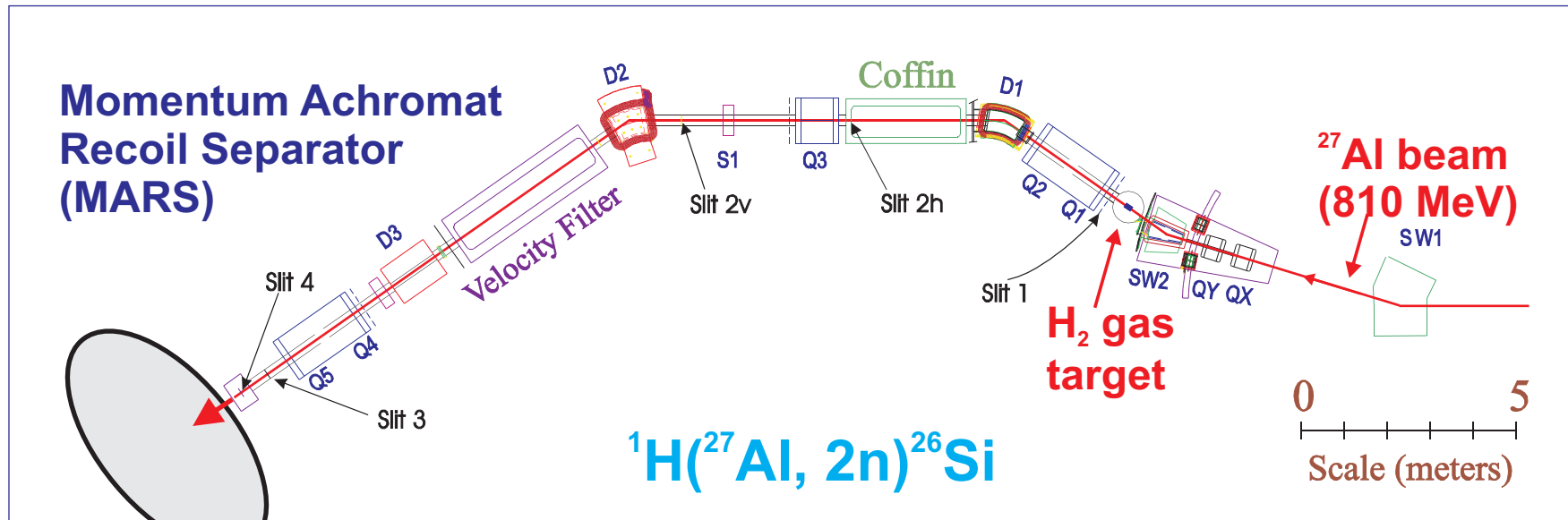
# PRECISION DECAY MEASUREMENTS AT TAMU



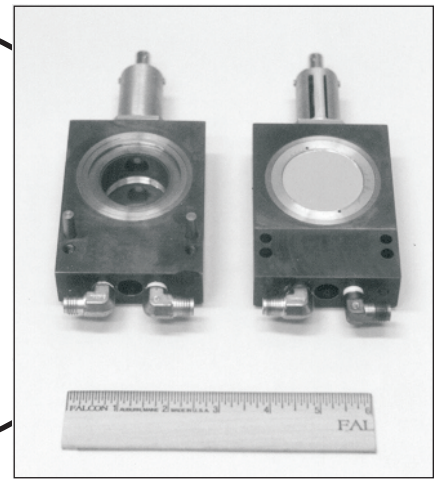
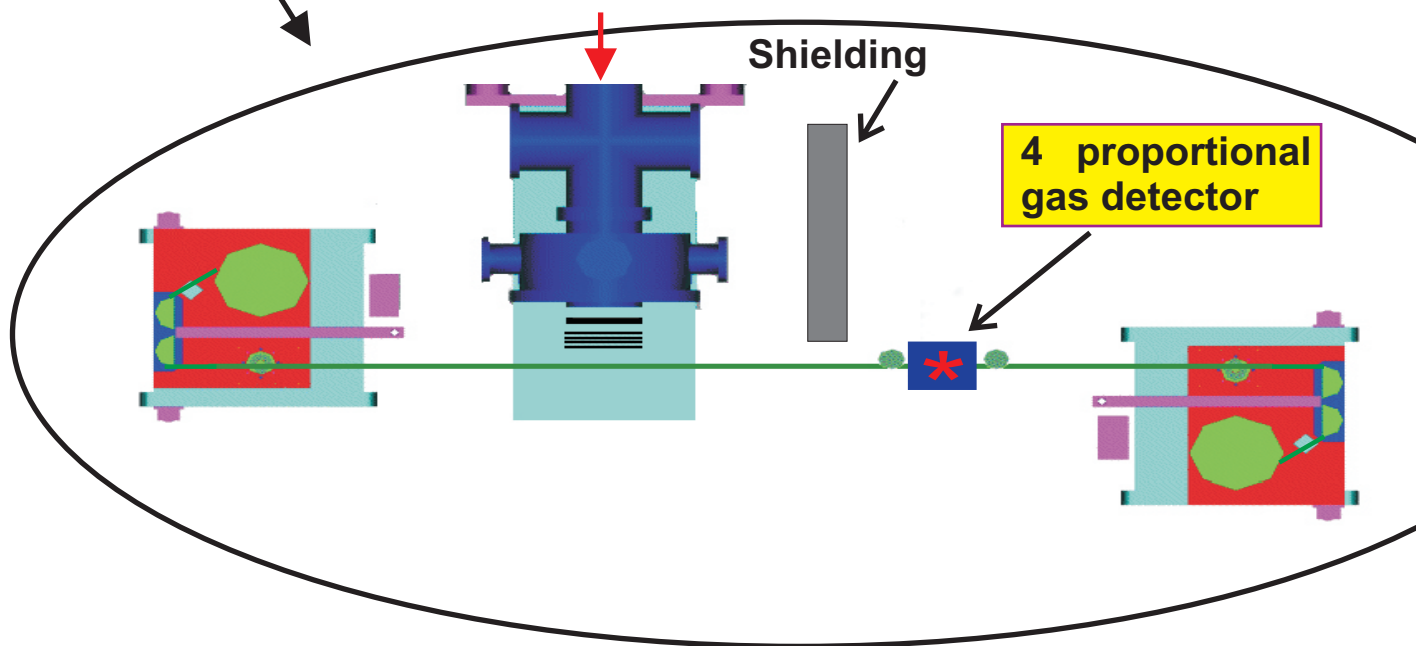
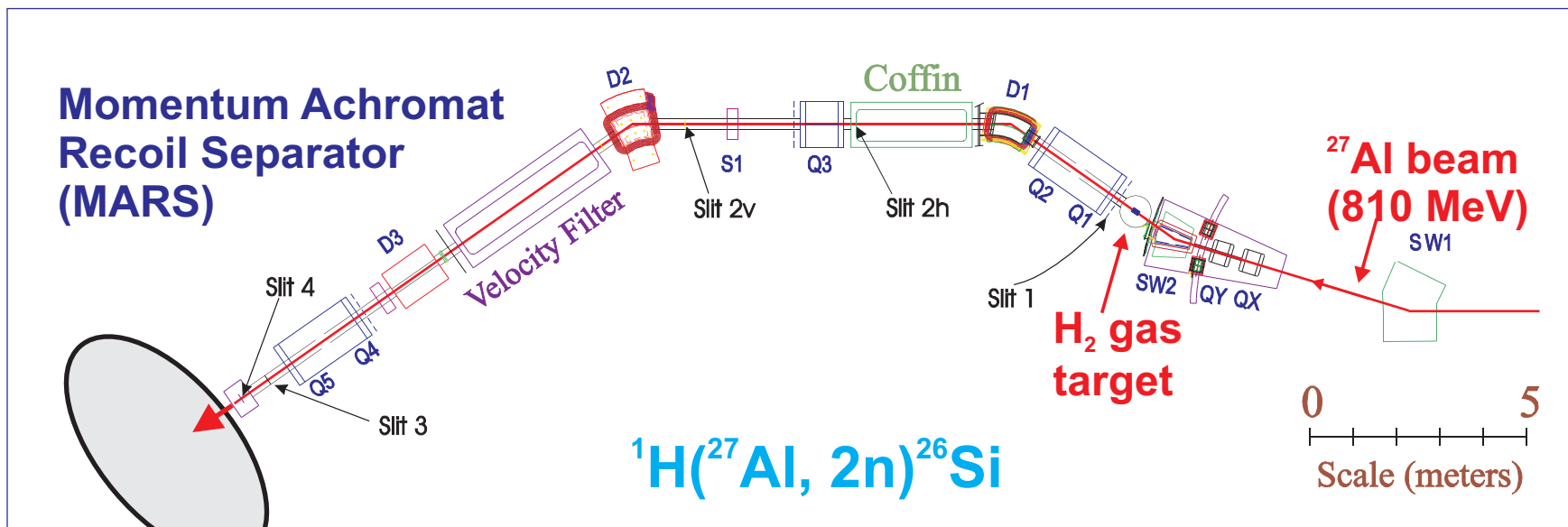
# PRECISION DECAY MEASUREMENTS AT TAMU



# PRECISION DECAY MEASUREMENTS AT TAMU



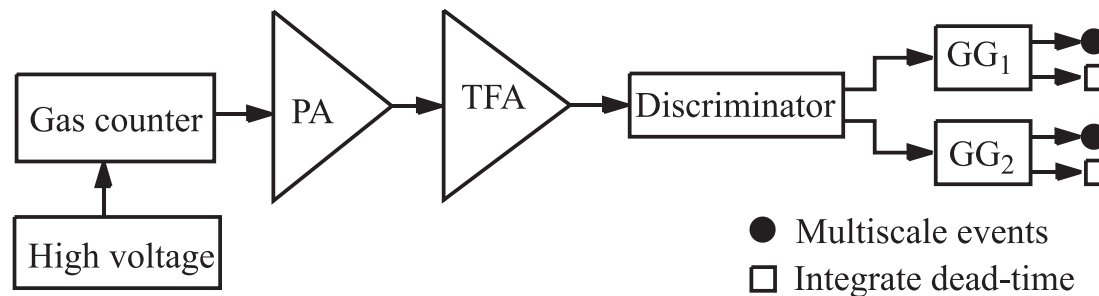
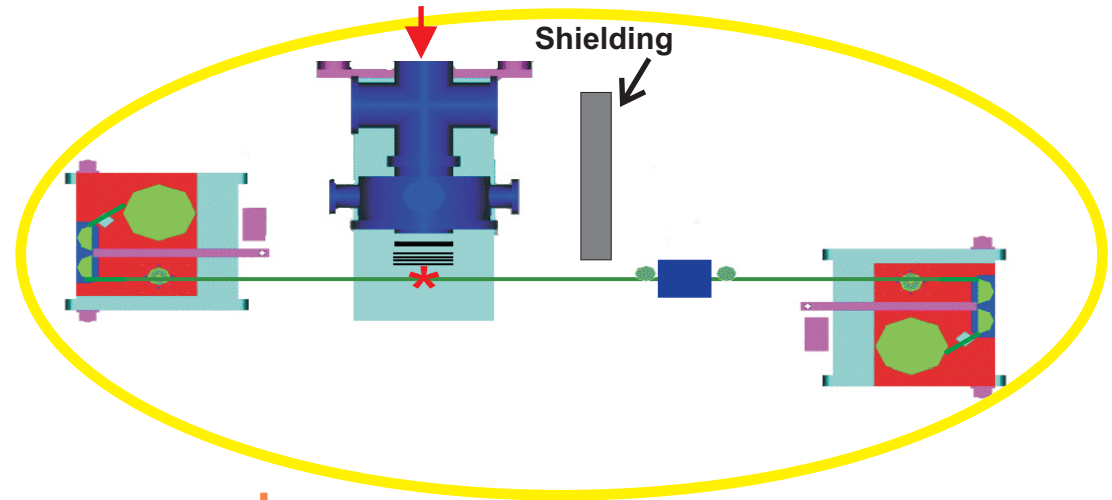
# PRECISION DECAY MEASUREMENTS AT TAMU



# REQUIREMENTS FOR PRECISE HALF-LIFE MEASUREMENT

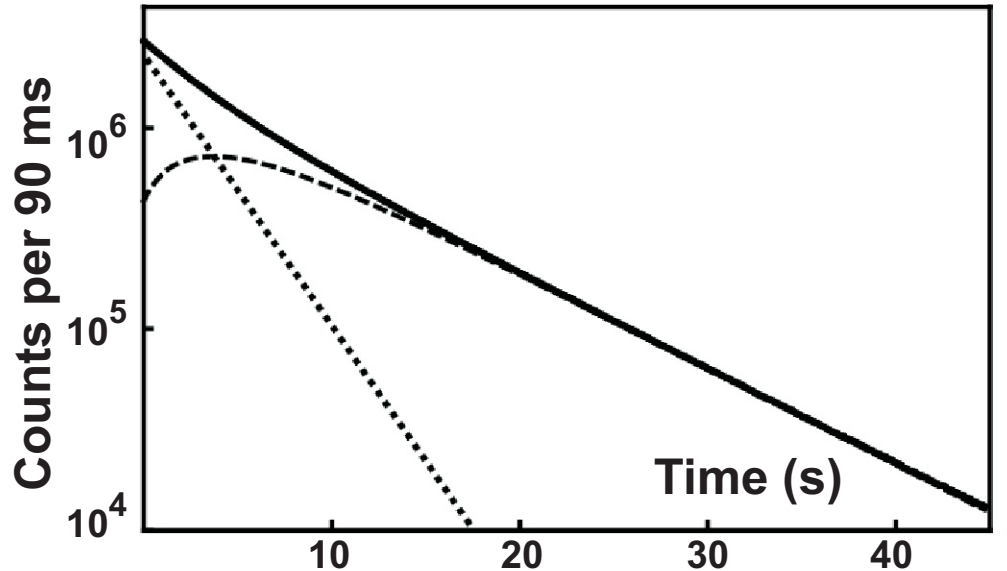
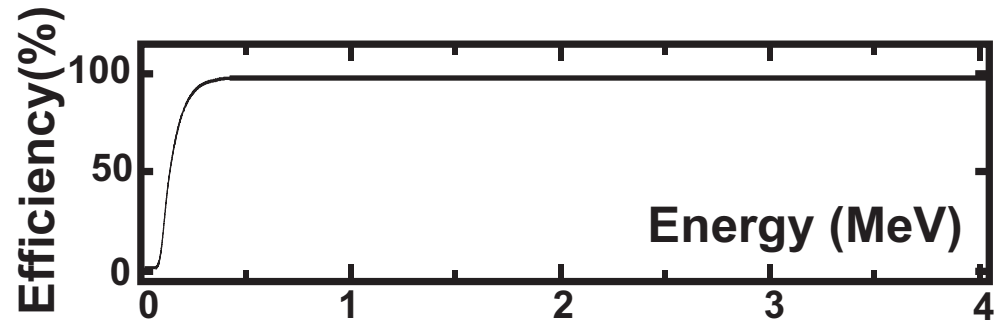
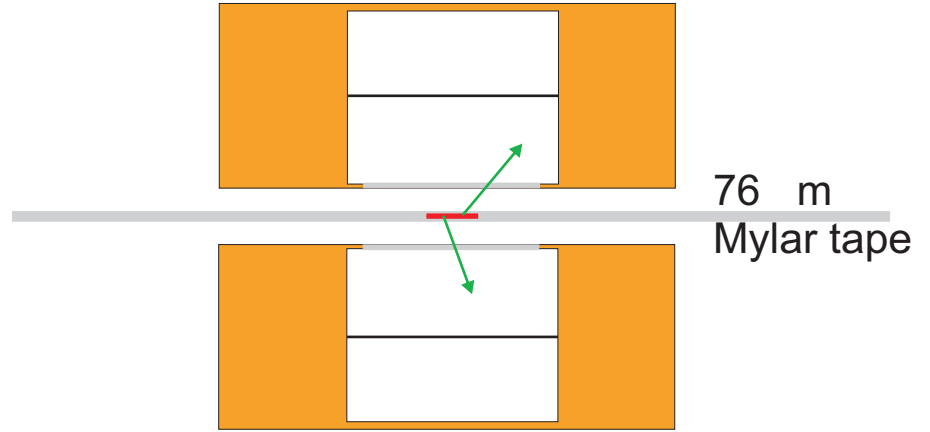
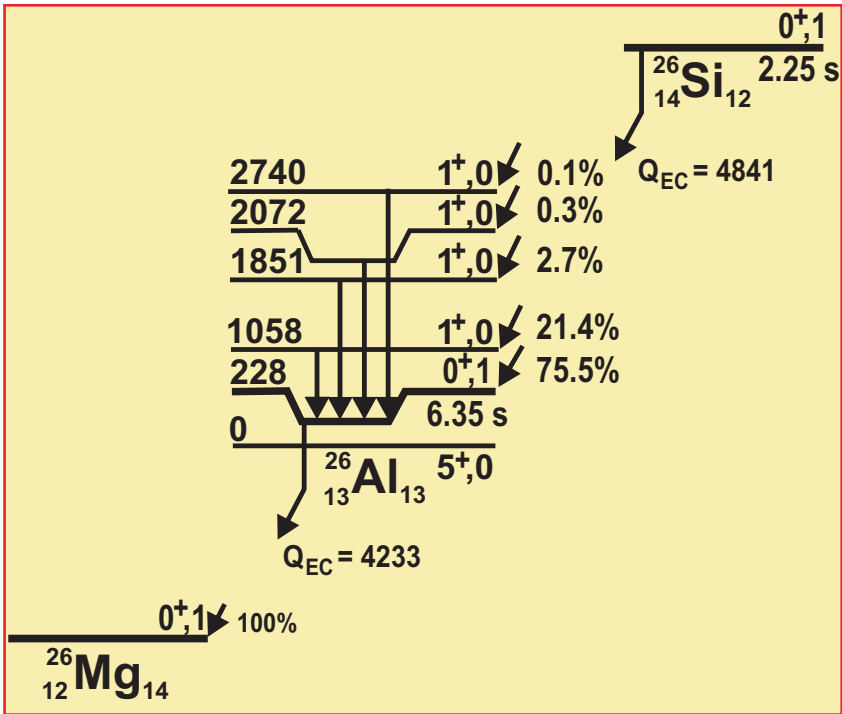
## IMPORTANT FEATURES

- Extremely high source purity -- separation by Z/A and range.
- Very low background
- Rapid transport (130 ms) to shielded counting position.
- Dominant dead-time, fixed and measured.



- Repeated measurements under different experimental conditions.
- Decay data stored cycle-by-cycle so actual instantaneous rate can be used in analysis.
- Precise statistical procedures used, including simultaneous fit to many cycles with single half-life.

# HALF LIFE OF $^{26}\text{Si}$



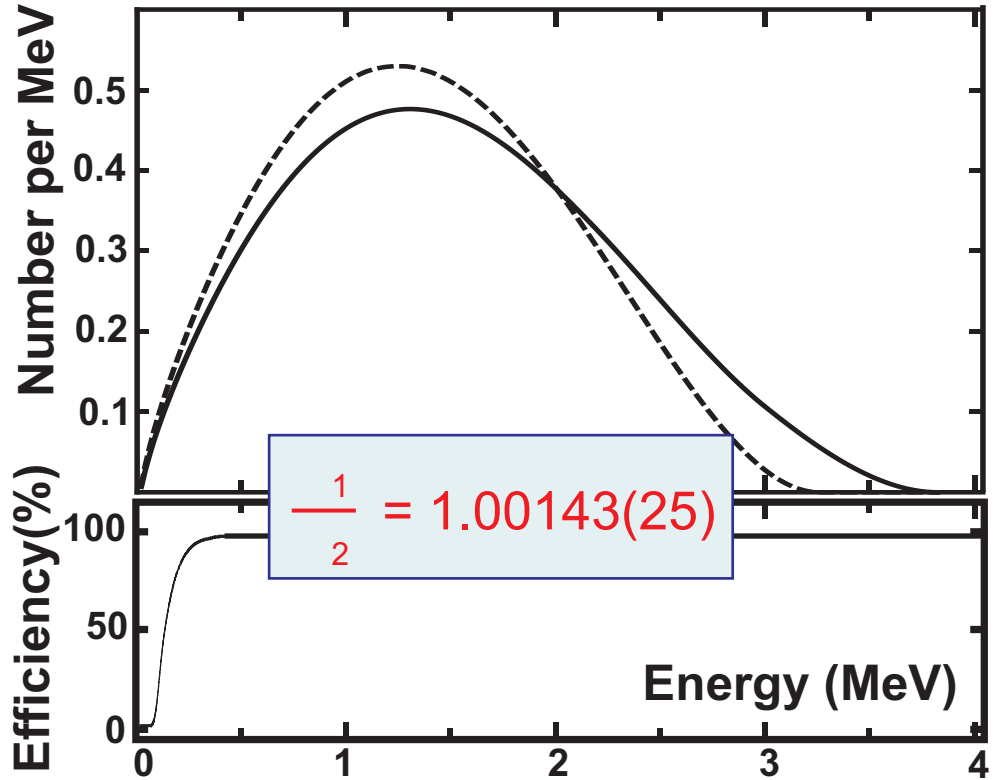
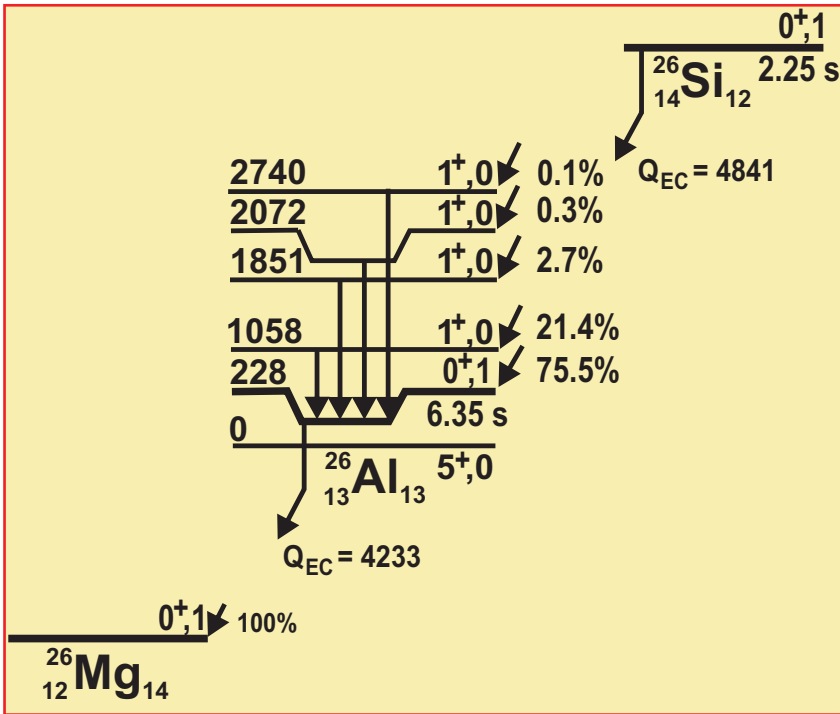
$$N_{\text{tot}} = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t}$$

where

$$C_1 = N_1 \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right)$$

$$C_2 = N_1 \left( \frac{\lambda_2}{\lambda_1 - \lambda_2} + \frac{1}{\lambda_1} \right)$$

# HALF LIFE OF $^{26}\text{Si}$

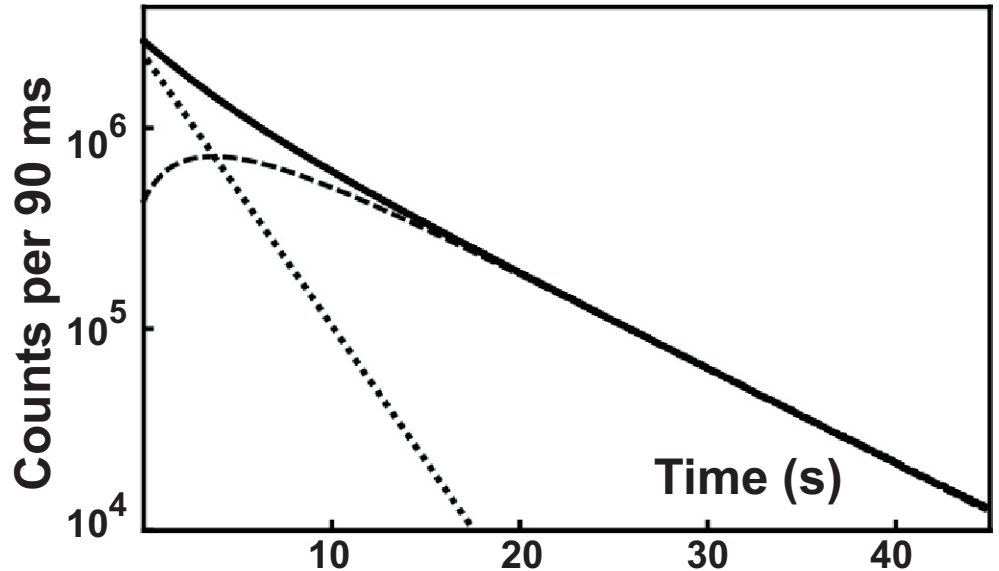


$$N_{\text{tot}} = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t}$$

where

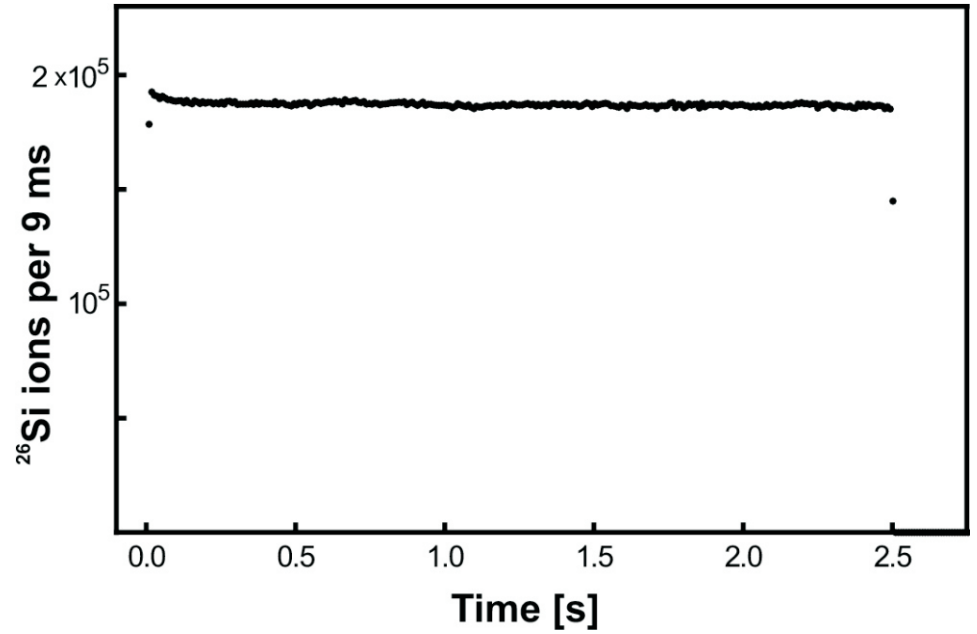
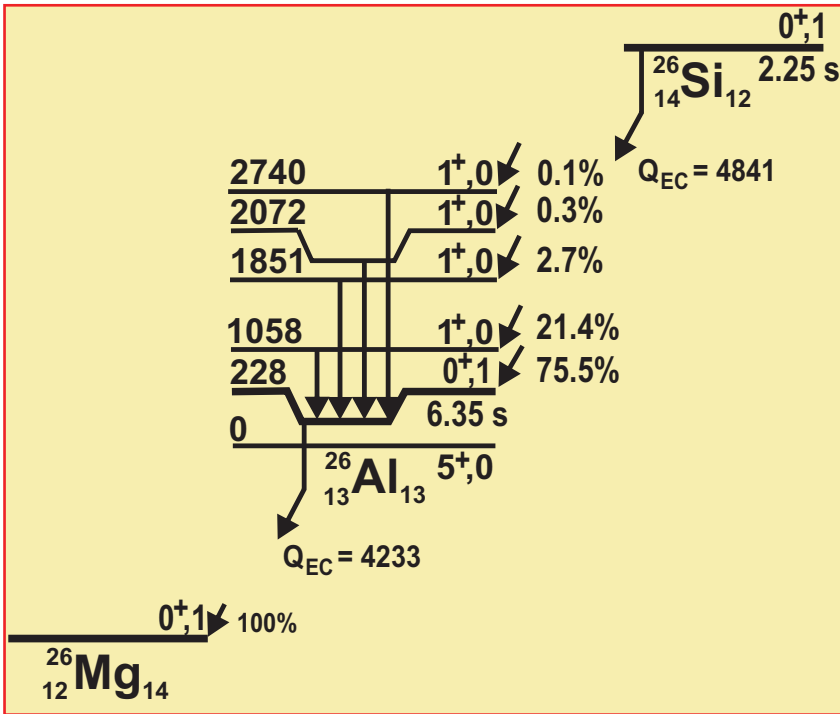
$$C_1 = N_1 \left( \frac{1}{2} - \frac{2}{1 - 2} \right)$$

$$C_2 = N_1 \left( \frac{N_2}{N_1} + \frac{1}{1 - 2} \right)$$





# HALF LIFE OF $^{26}\text{Si}$

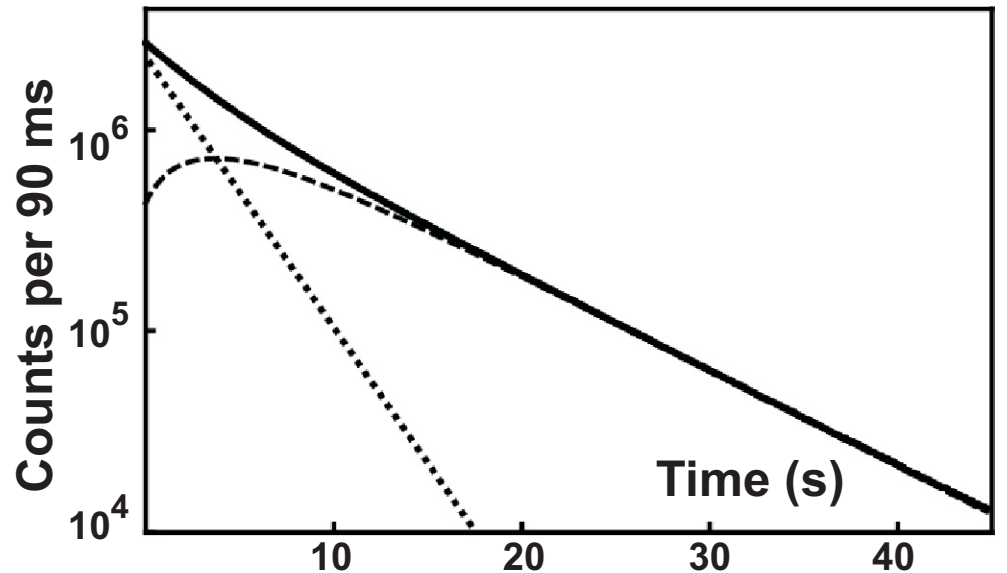


$$N_{\text{tot}} = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t}$$

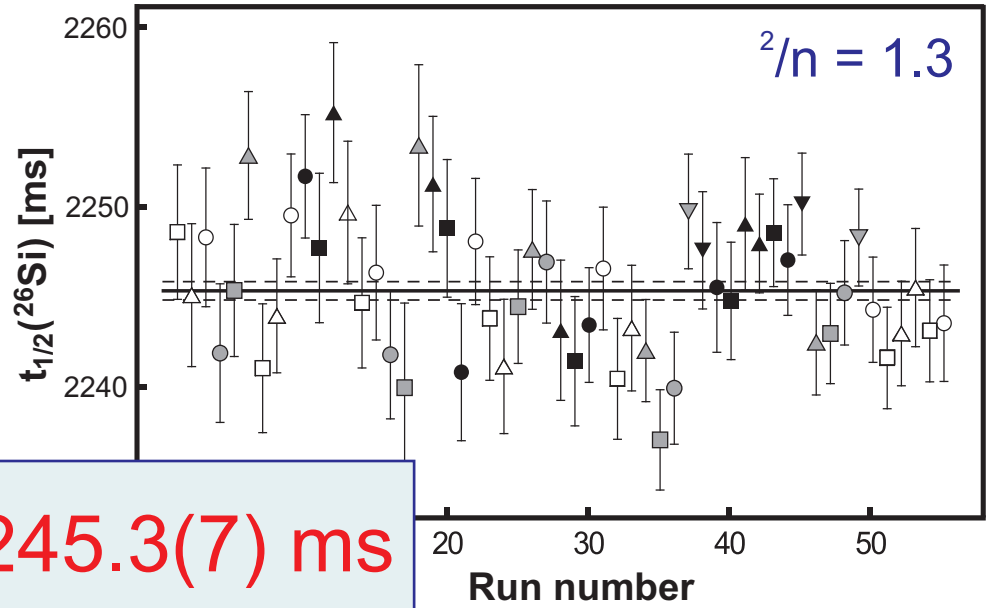
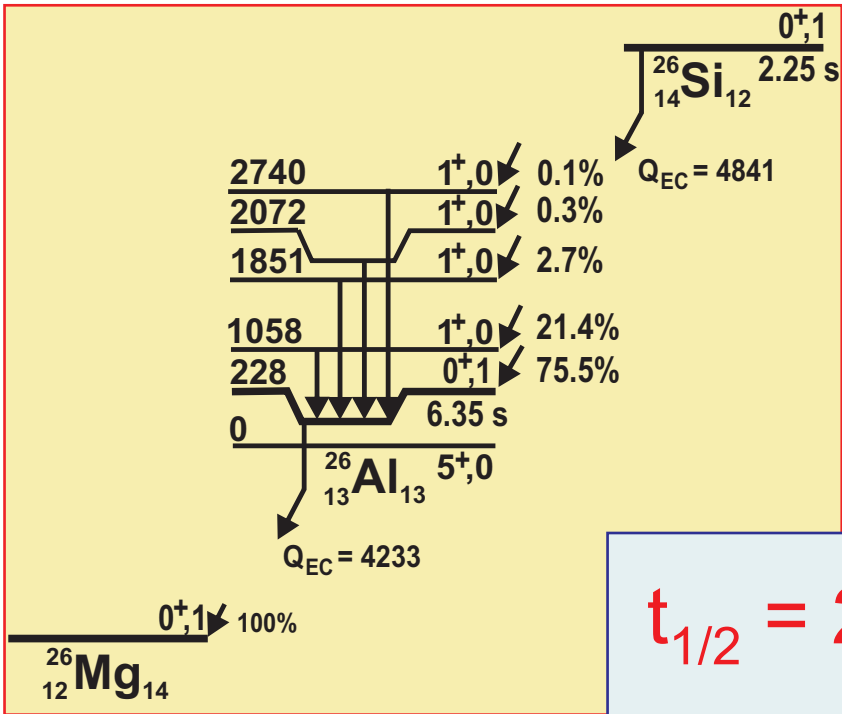
where

$$C_1 = N_1 \left( \frac{1}{2} - \frac{2}{1 - 2} \right)$$

$$C_2 = N_1 \left( \frac{N_2}{N_1} + \frac{1}{1 - 2} \right)$$



# HALF LIFE OF $^{26}\text{Si}$



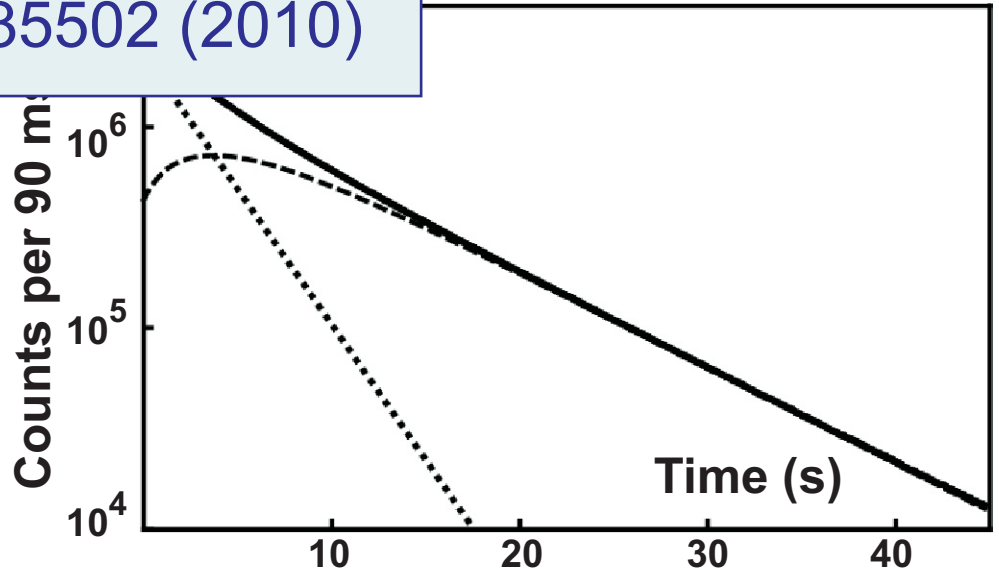
$t_{1/2} = 2245.3(7) \text{ ms}$   
 Iacob et al., PRC 82, 035502 (2010)

$$N_{\text{tot}} = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t}$$

where

$$C_1 = N_1 \left( \frac{1}{2} - \frac{2}{1 - 2} \right)$$

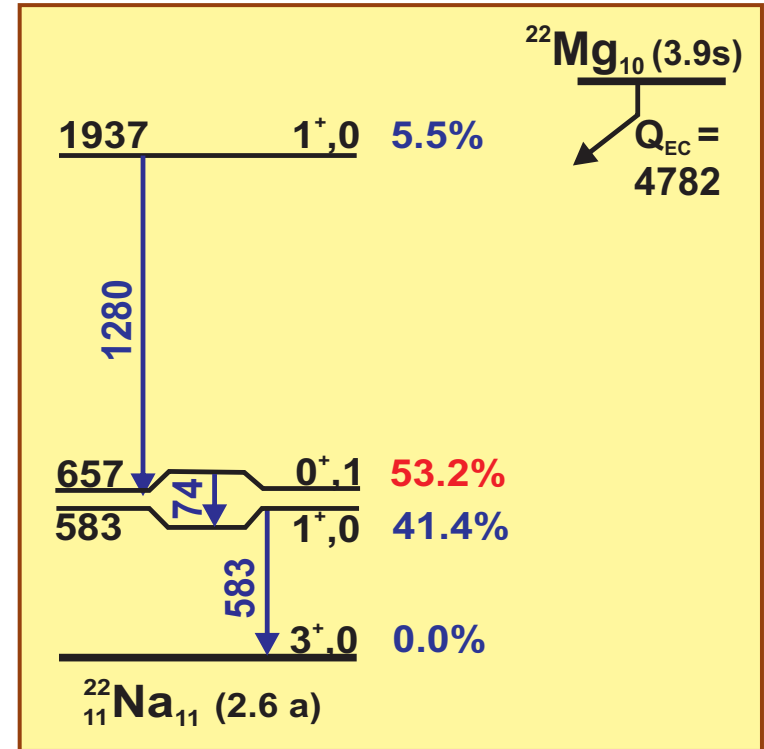
$$C_2 = N_1 \left( \frac{N_2}{N_1} + \frac{1}{1 - 2} \right)$$



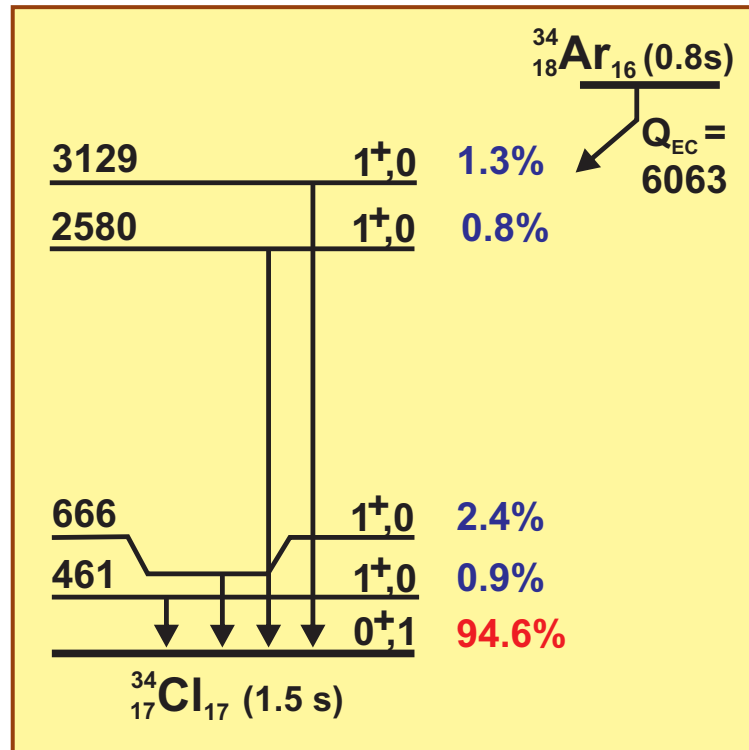
# BRANCHING-RATIO MEASUREMENTS

In all cases we measure the intensities of  $\beta^-$ -delayed  $\gamma$  rays.

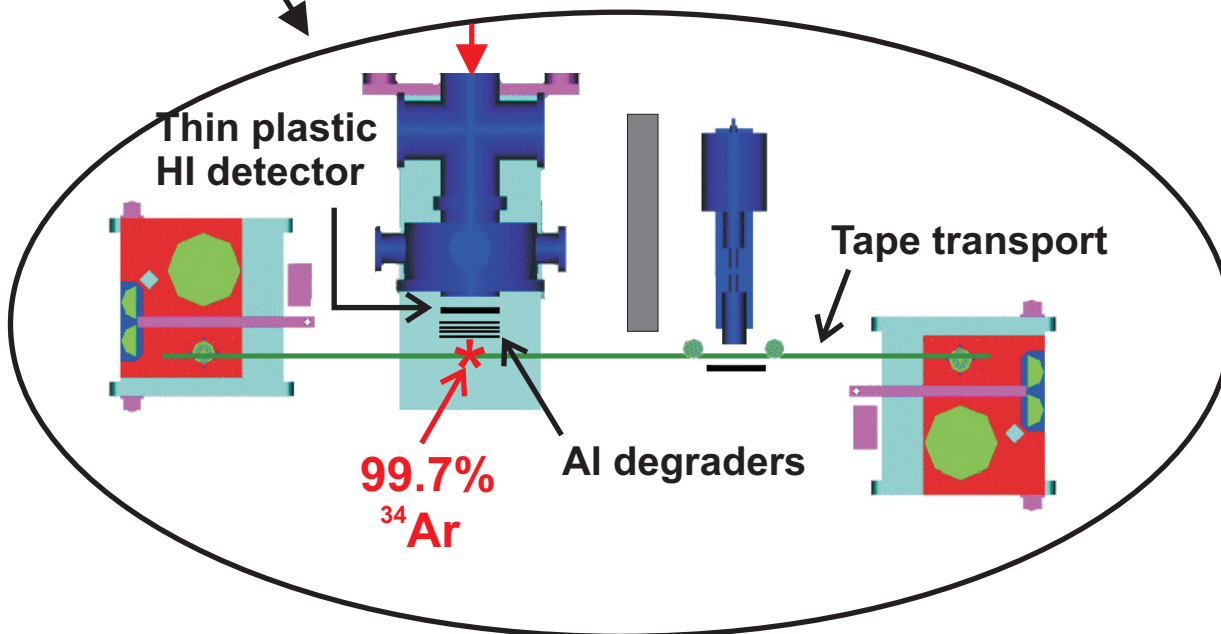
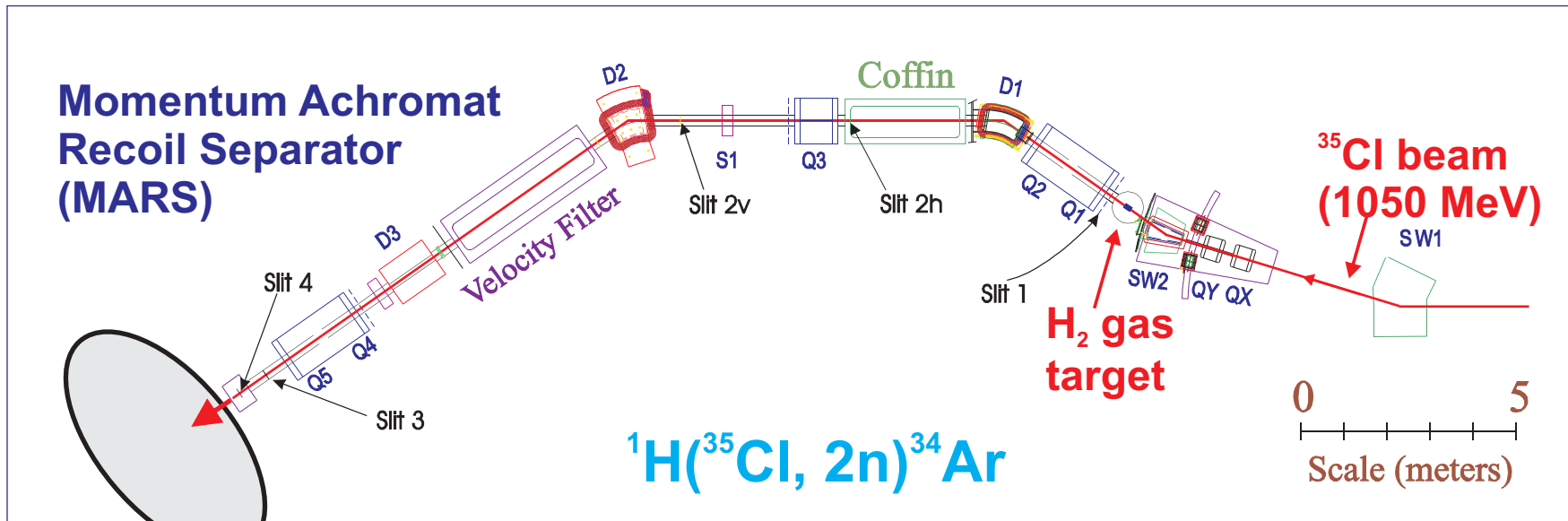
Relative intensities



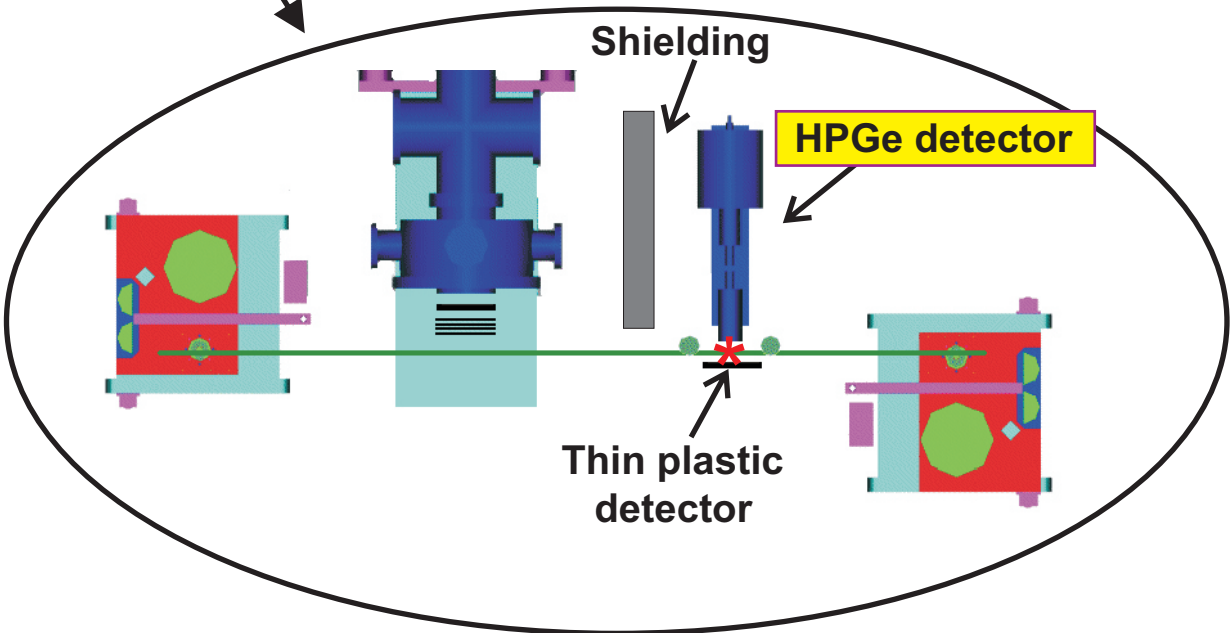
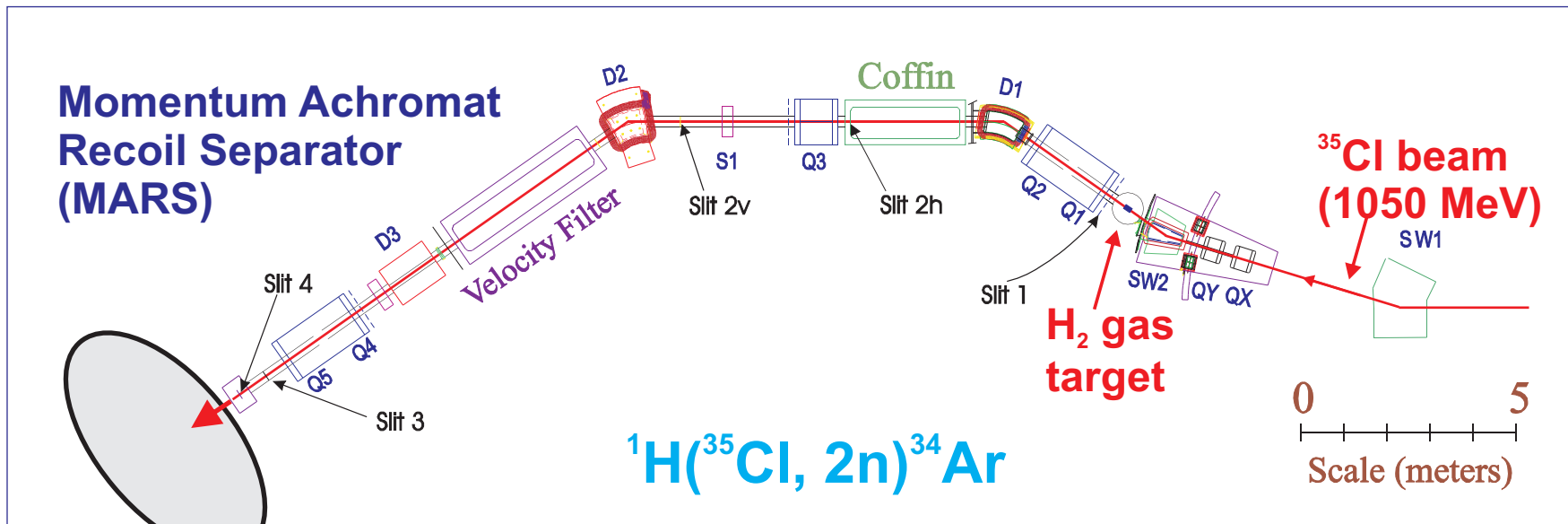
Absolute intensities



# PRECISION DECAY MEASUREMENTS AT TAMU



# PRECISION DECAY MEASUREMENTS AT TAMU



HPGe detector calibrated for efficiency to  $\pm 0.2\%$

# HPGe DETECTOR CALIBRATION

## Commercial standard sources:

Relative intensities not known in any case to better than 0.4%.

Source activity (absolute intensity) can be specified to 2-5%; rarely to 1%.

## For higher precision:

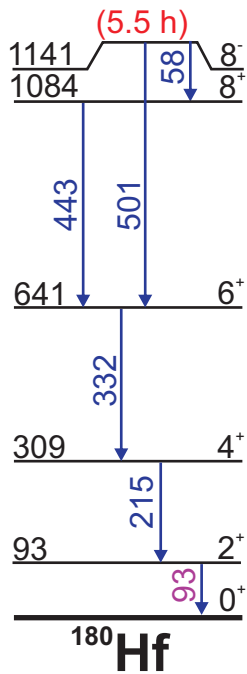
Source activity for certain cases can be measured to 0.1% by 4 coincidence counting; in our case  $^{60}\text{Co}$  at PTB Lab.

Schoenfeld *et al.*,  
Appl. Rad & Isot.  
56 (2002) 215.

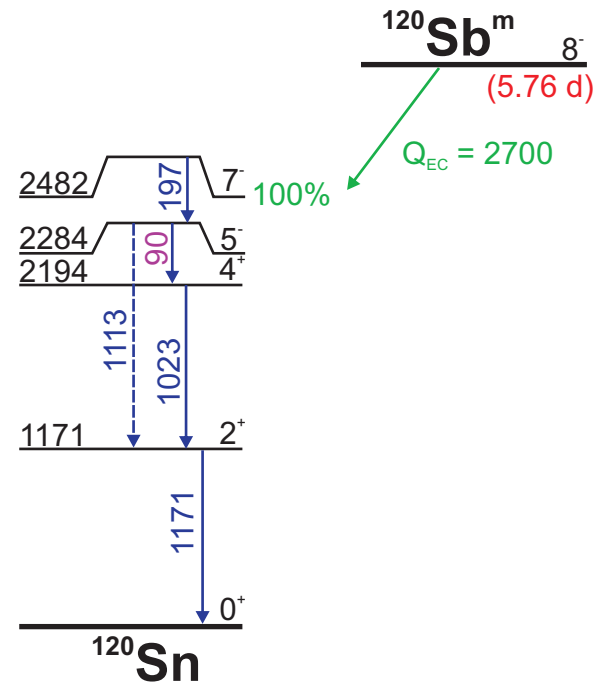
Use clean  $\gamma$ -ray cascades; home-made sources.

Combine Monte Carlo calculations with measured points.

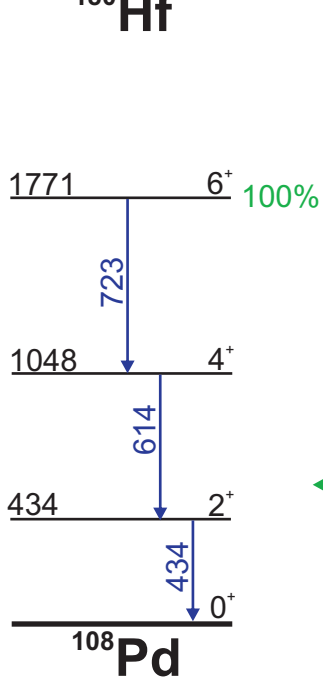
# KEY RADIOACTIVE SOURCES



$^{120}\text{Sn} (p,n) ^{120}\text{Sb}^m$  at TAMU cyclotron

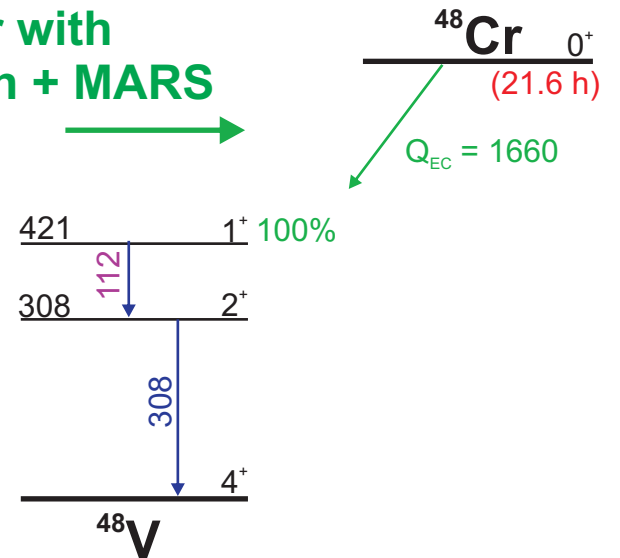


$^{179}\text{Hf} (n, \gamma) ^{180}\text{Hf}$  at TAMU reactor



$^{108}\text{Ag}^m$   $6^+$  (418 a)

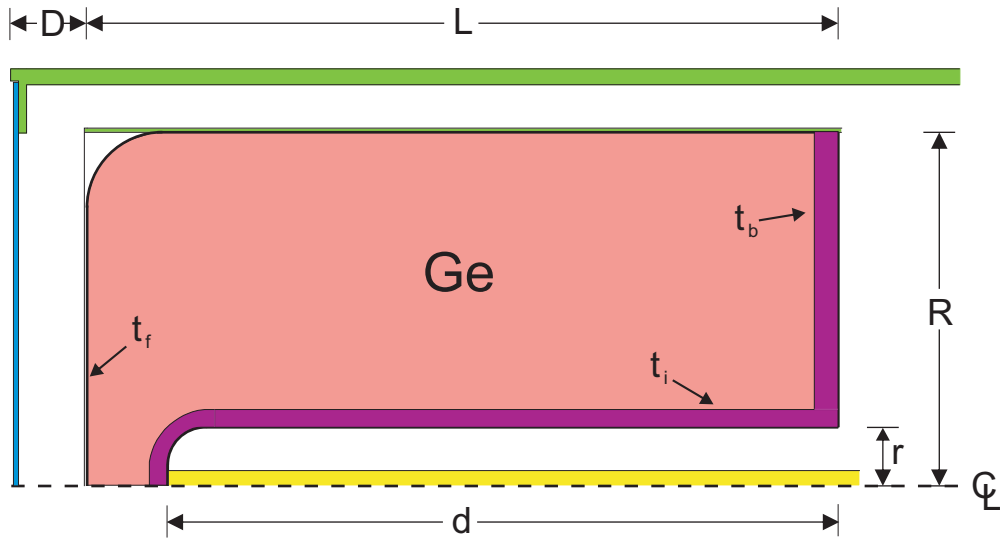
$^1\text{H} (^{50}\text{Cr}, p2n) ^{48}\text{Cr}$  with TAMU cyclotron + MARS



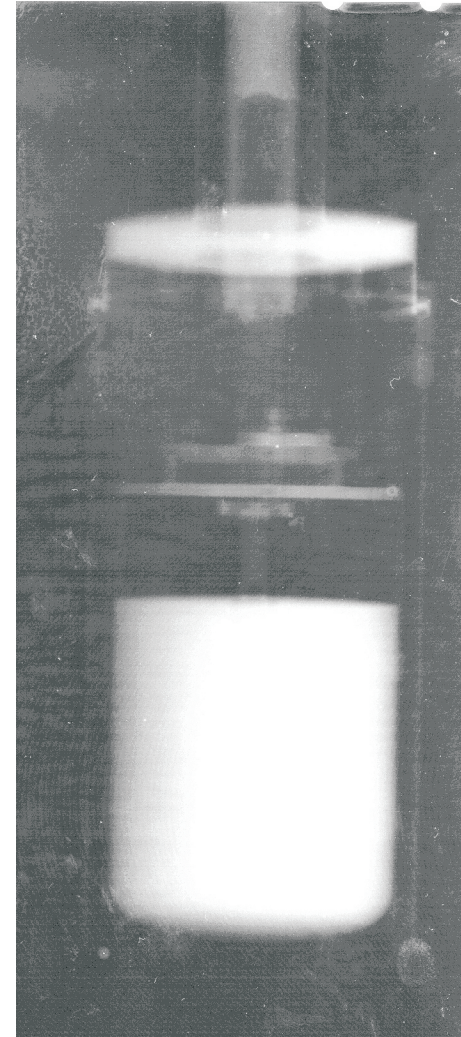
Impurity in commercial  $^{110}\text{Ag}^m$  source

# MONTE CARLO CALCULATIONS

## EG&G ORTEC Gamma-X HPGe



X-ray picture of crystal

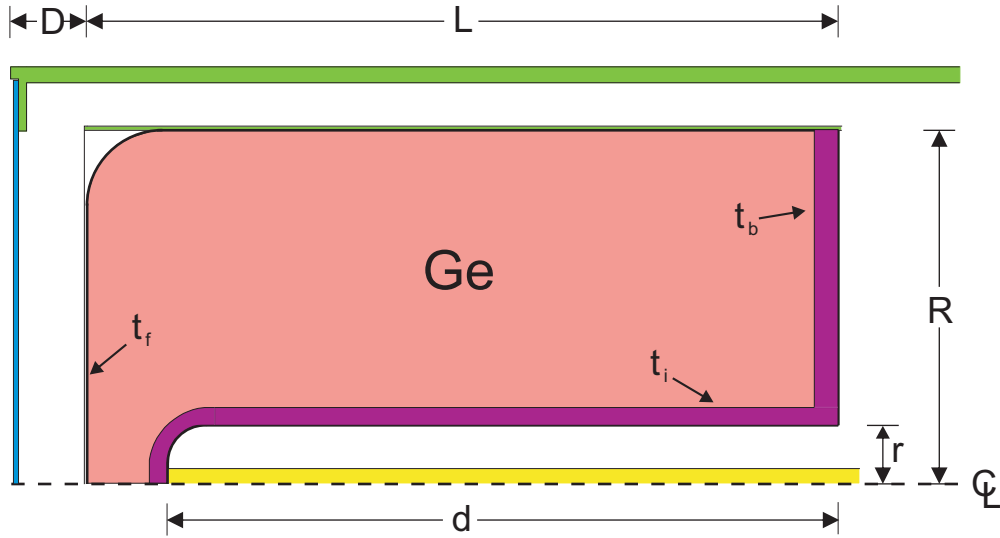


| DIMENSION                              | NOMINAL  |
|--|----------|
| Crystal radius, R                      | 34.95 mm |
| Crystal active length, $L - t_f - t_b$ | 77.7 mm  |
| Cap face to crystal distance, D        | 5.6 mm   |
| Hole radius, r                         | 5.8 mm   |
| Hole depth, d                          | 69.7 mm  |
| Depth internal (Li) dead layer, $t_i$  | >1 mm    |
| Depth front dead layer, $t_f$          | >0.3 m   |



# MONTE CARLO CALCULATIONS

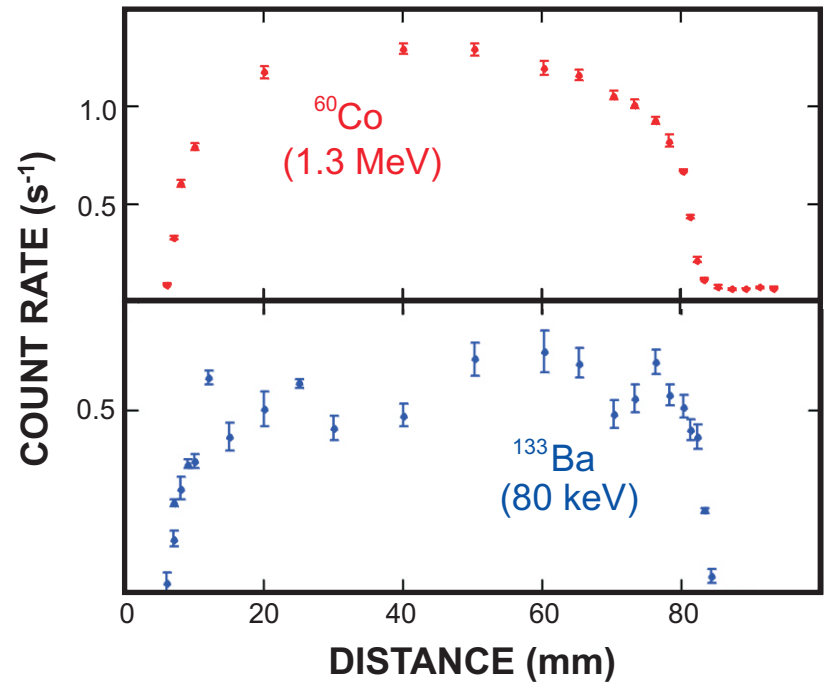
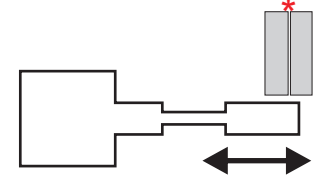
## EG&G ORTEC Gamma-X HPGe



| DIMENSION                              | NOMINAL  | MEASURED<br>or FITTED |
|--|----------|-----------------------|
| Crystal radius, $R$                    | 34.95 mm |                       |
| Crystal active length, $L - t_f - t_b$ | 77.7 mm  | 75.4 mm               |
| Cap face to crystal distance, $D$      | 5.6 mm   |                       |
| Hole radius, $r$                       | 5.8 mm   |                       |
| Hole depth, $d$                        | 69.7 mm  |                       |
| Depth internal (Li) dead layer, $t_i$  | >1 mm    |                       |
| Depth front dead layer, $t_f$          | >0.3 mm  |                       |

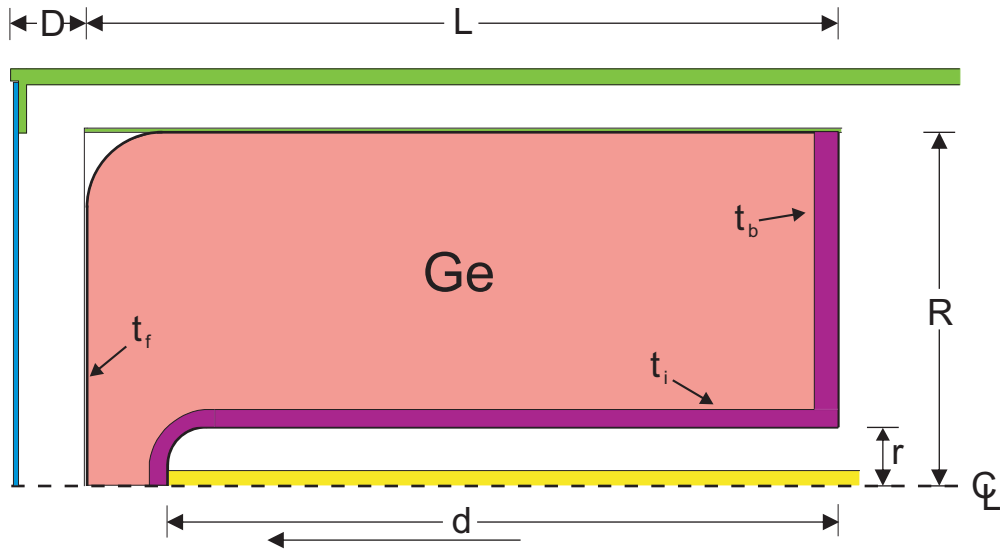
## X-ray picture of crystal

### Crystal side-scan



# MONTE CARLO CALCULATIONS

## EG&G ORTEC Gamma-X HPGe



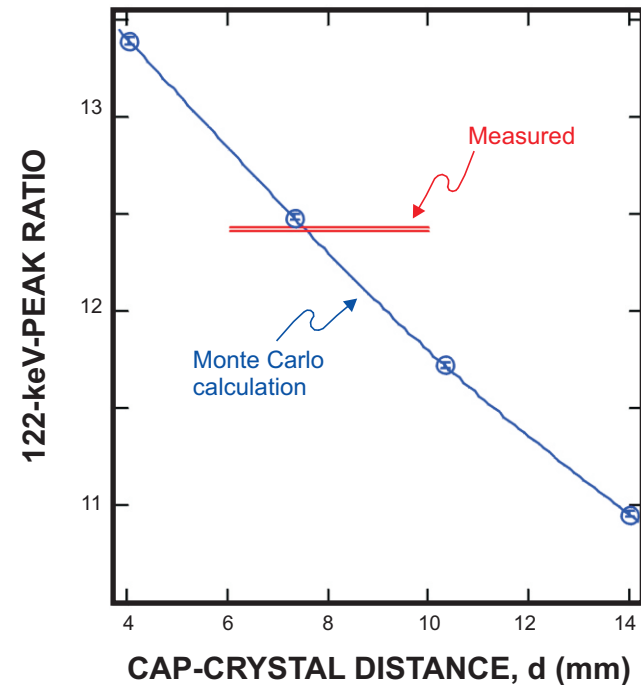
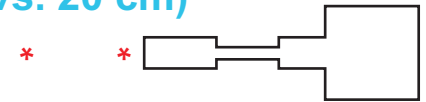
| DIMENSION                              | NOMINAL  | MEASURED<br>or FITTED |
|--|----------|-----------------------|
| Crystal radius, R                      | 34.95 mm |                       |
| Crystal active length, $L - t_f - t_b$ | 77.7 mm  | 75.4 mm               |
| Cap face to crystal distance, D        | 5.6 mm   | 7.2 mm                |
| Hole radius, r                         | 5.8 mm   |                       |
| Hole depth, d                          | 69.7 mm  |                       |
| Depth internal (Li) dead layer, $t_i$  | >1 mm    |                       |
| Depth front dead layer, $t_f$          | >0.3 mm  |                       |

X-ray picture of crystal

Crystal side-scan

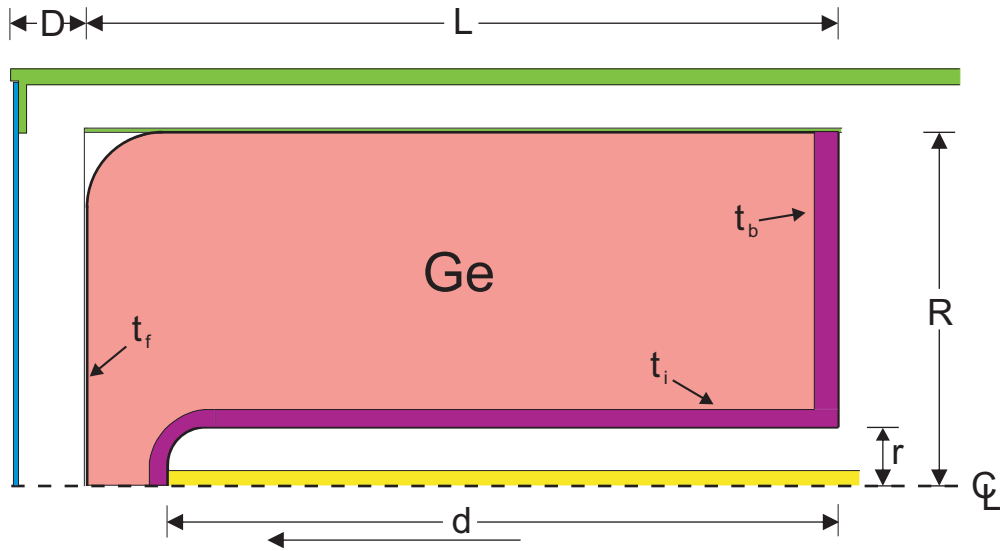
Distance ratio for  $^{57}\text{Co}$

(4 cm vs. 20 cm)



# MONTE CARLO CALCULATIONS

## EG&G ORTEC Gamma-X HPGe



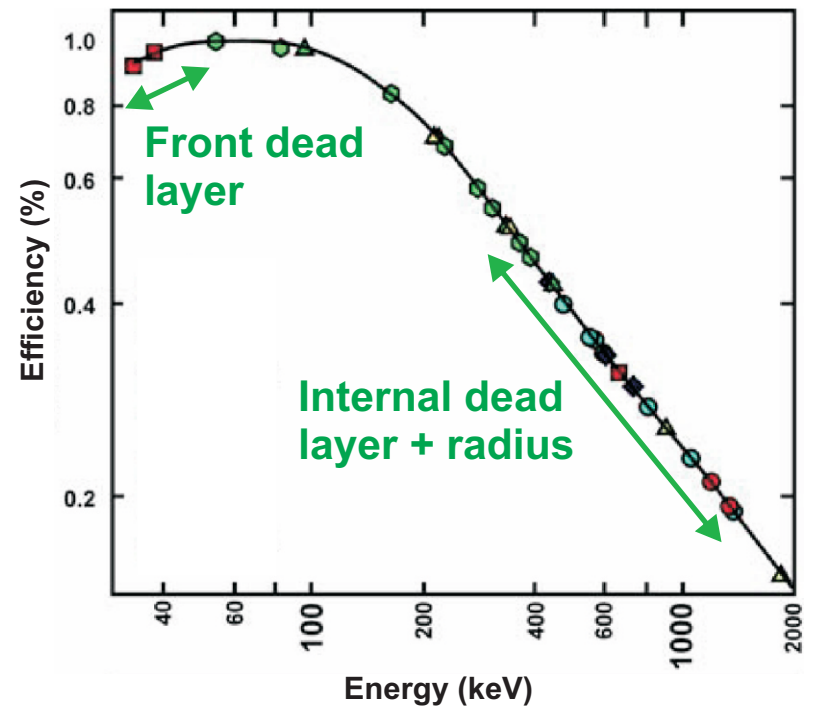
X-ray picture of crystal

Crystal side-scan

Distance ratio for <sup>57</sup>Co

Fitted for energy dependence

| DIMENSION                              | NOMINAL  | MEASURED<br>or FITTED |
|--|----------|-----------------------|
| Crystal radius, R                      | 34.95 mm | 34.49 mm              |
| Crystal active length, $L - t_f - t_b$ | 77.7 mm  | 75.4 mm               |
| Cap face to crystal distance, D        | 5.6 mm   | 7.2 mm                |
| Hole radius, r                         | 5.8 mm   |                       |
| Hole depth, d                          | 69.7 mm  |                       |
| Depth internal (Li) dead layer, $t_i$  | >1 mm    | 1.34 mm               |
| Depth front dead layer, $t_f$          | >0.3 m   | 2.5 m                 |



# DETECTOR EFFICIENCY

50 keV < E < 1.4 MeV

## Source measurements

VS

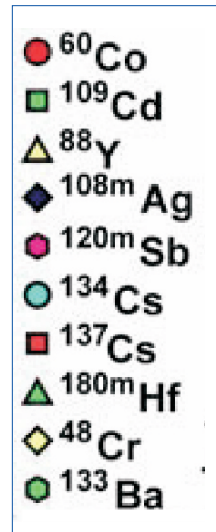
## unscaled Monte Carlo calculations (CYLTRAN)

Physical properties and location of HPGe crystal measured precisely

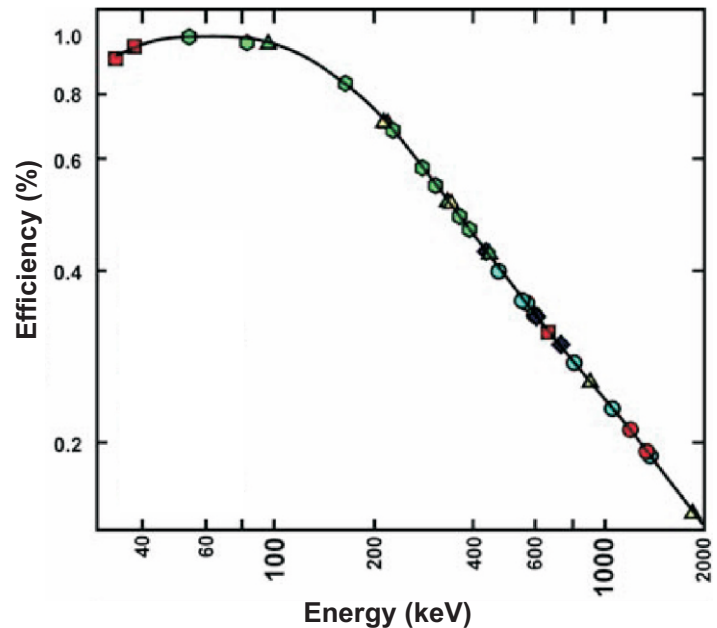
10 sources recorded

4 key sources, 3 locally made, have pure cascades

$^{60}\text{Co}$  source from PTB with activity known to  $\pm 0.1\%$



Helmer *et al.*,  
NIM A511, 360 (2003)



# DETECTOR EFFICIENCY

## 50 keV < E < 1.4 MeV

Source measurements

VS

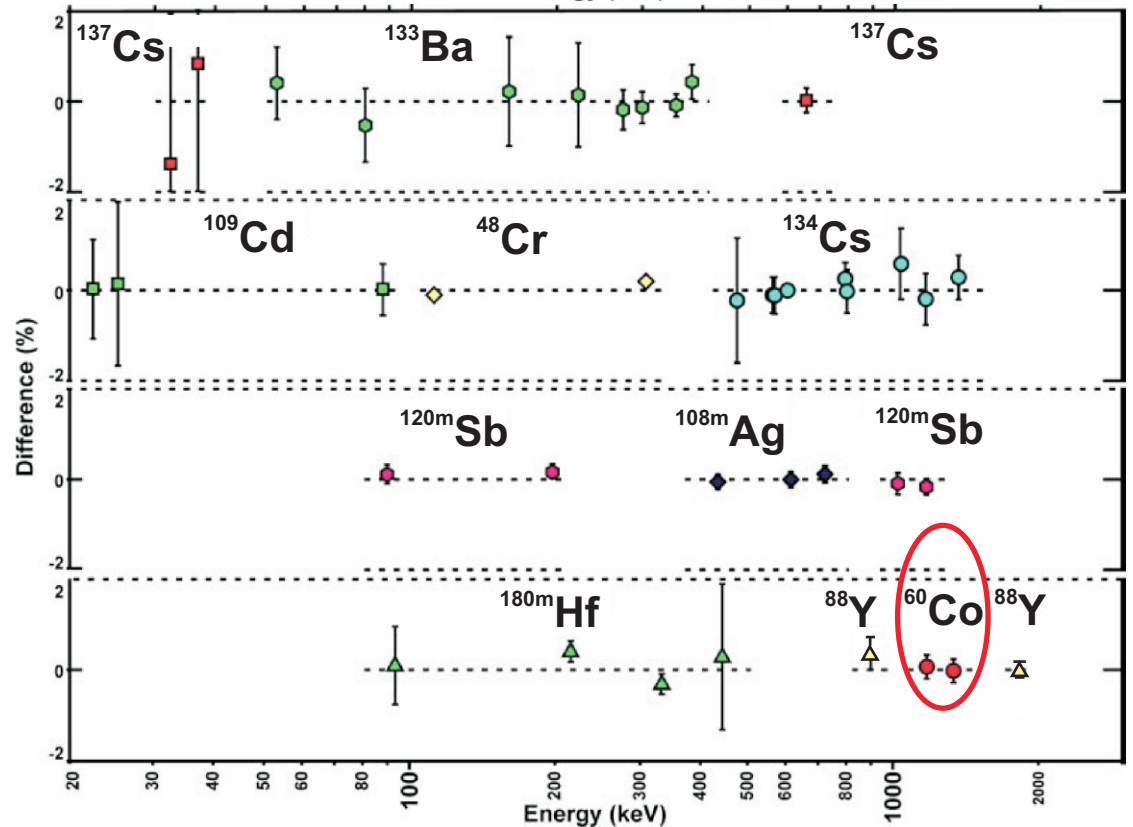
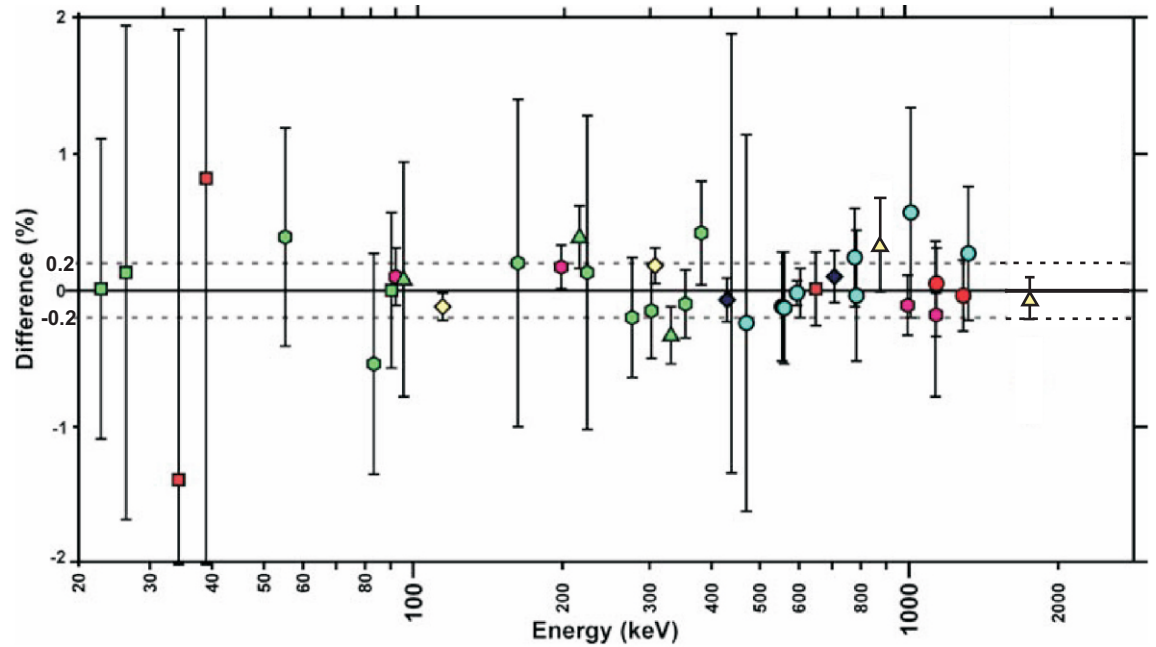
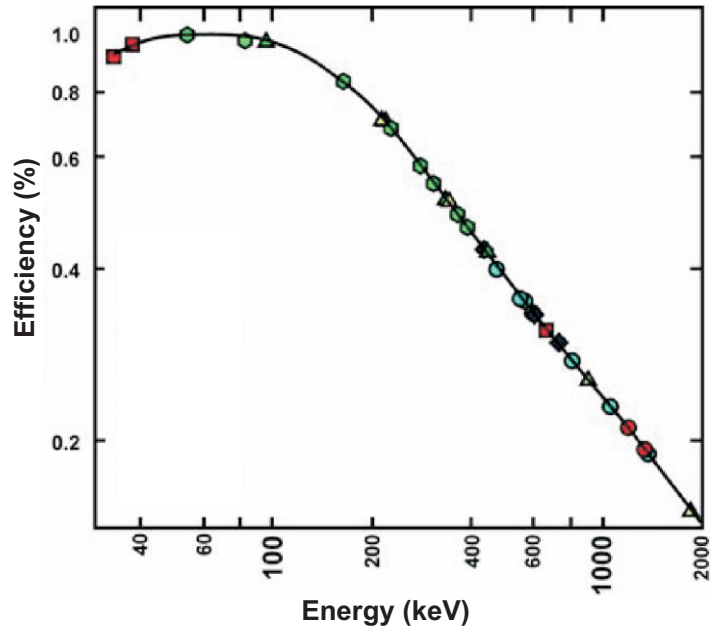
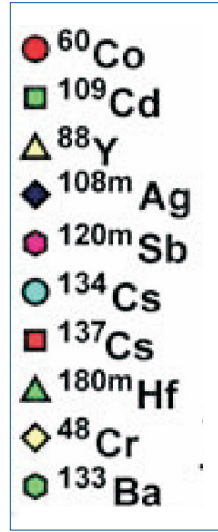
unscaled Monte Carlo  
calculations (CYLTRAN)

Physical properties and  
location of HPGe crystal  
measured precisely

10 sources recorded

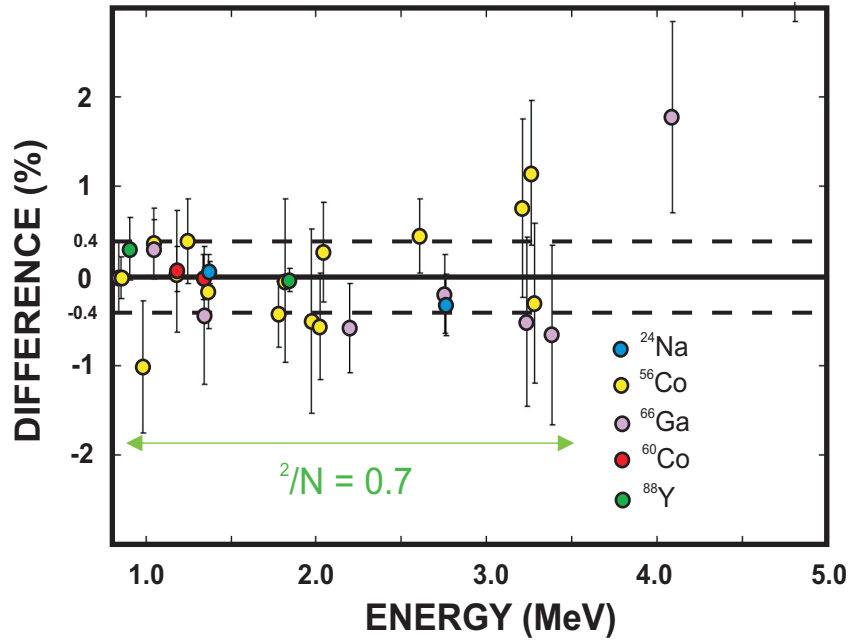
4 key sources, 3 locally  
made, have pure cascades

<sup>60</sup>Co source from PTB with  
activity known to ± 0.1%



# DETECTOR CHARACTERIZATION - DETAILS

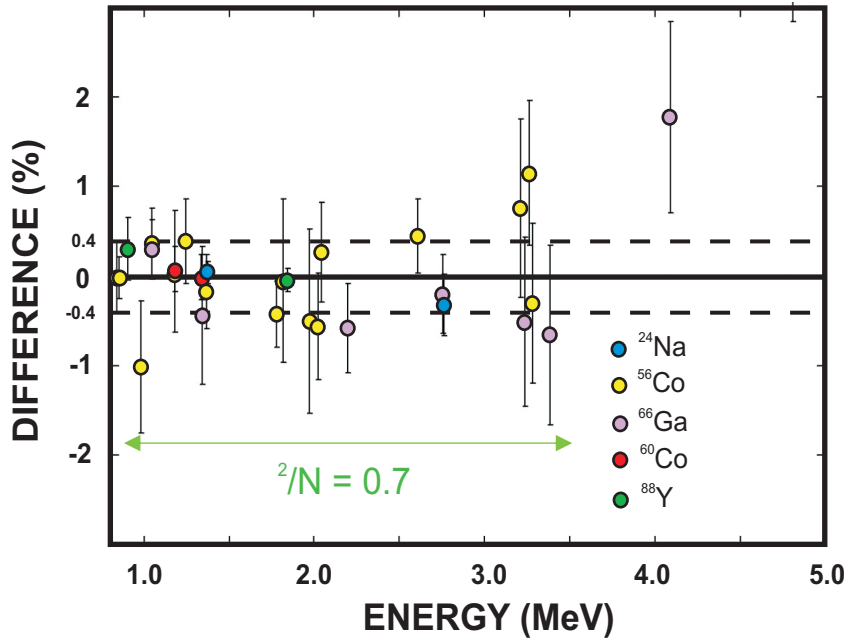
Efficiency extended up to 3.5 MeV



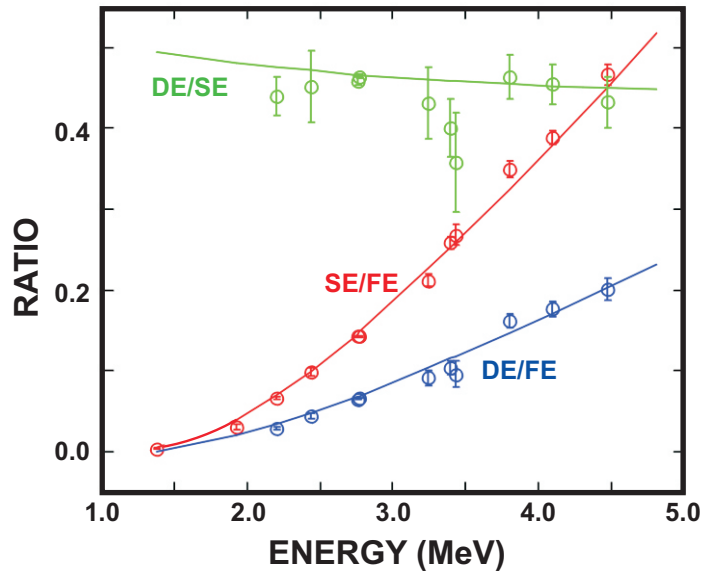
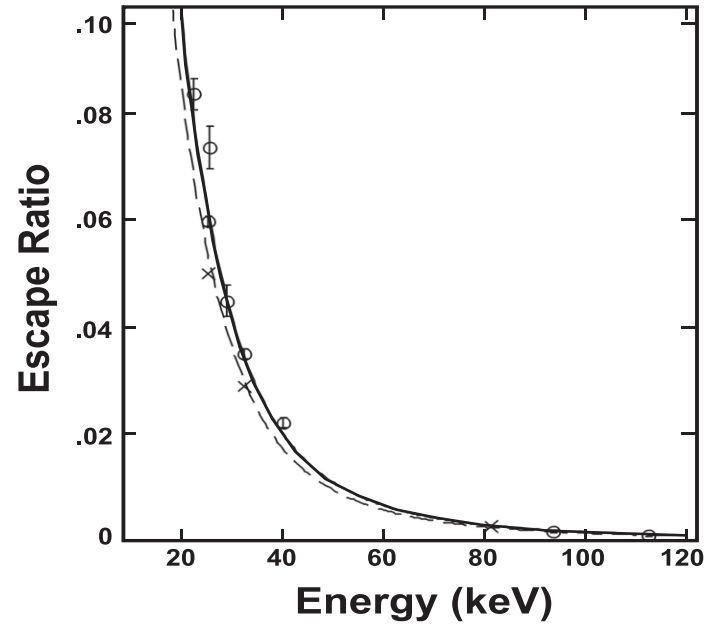
Helmer et al., Appl. Rad. Isot. 60, 173 (2004).

# DETECTOR CHARACTERIZATION - DETAILS

Efficiency extended up to 3.5 MeV

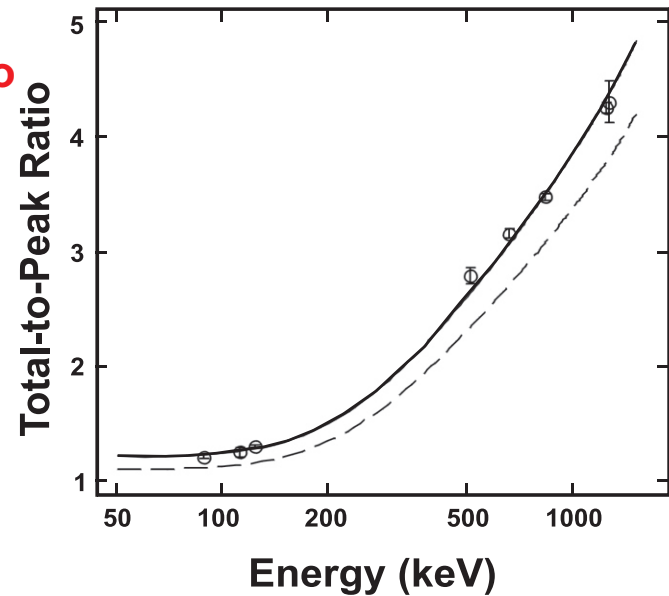


Ge x-ray escape

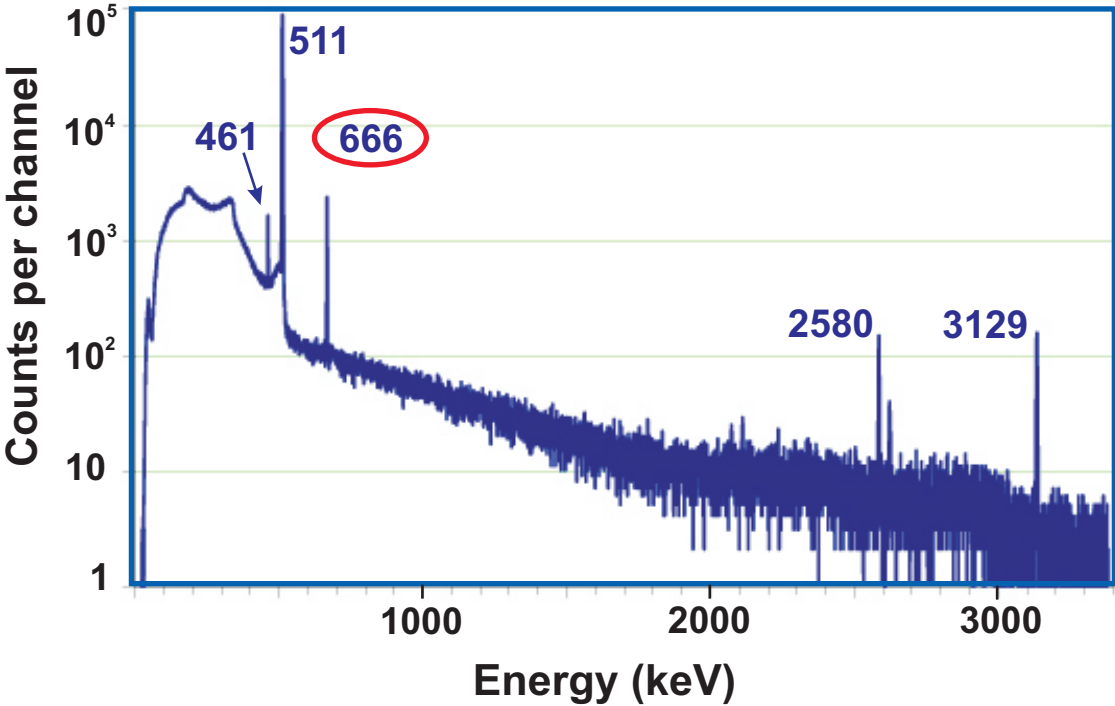
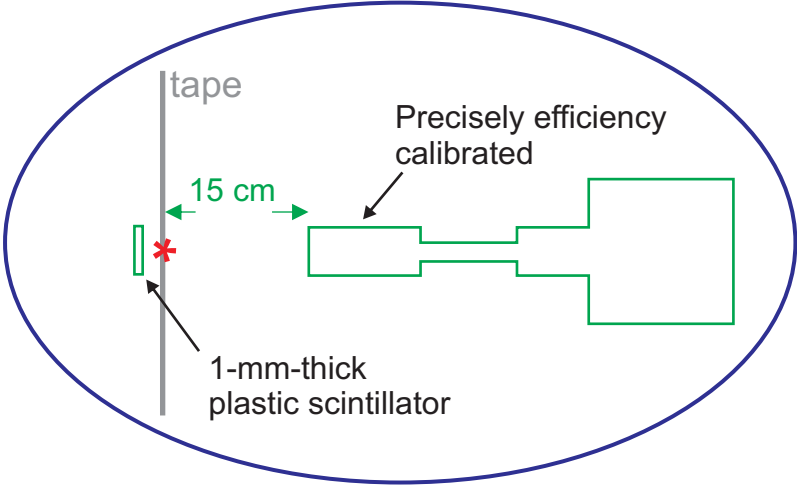
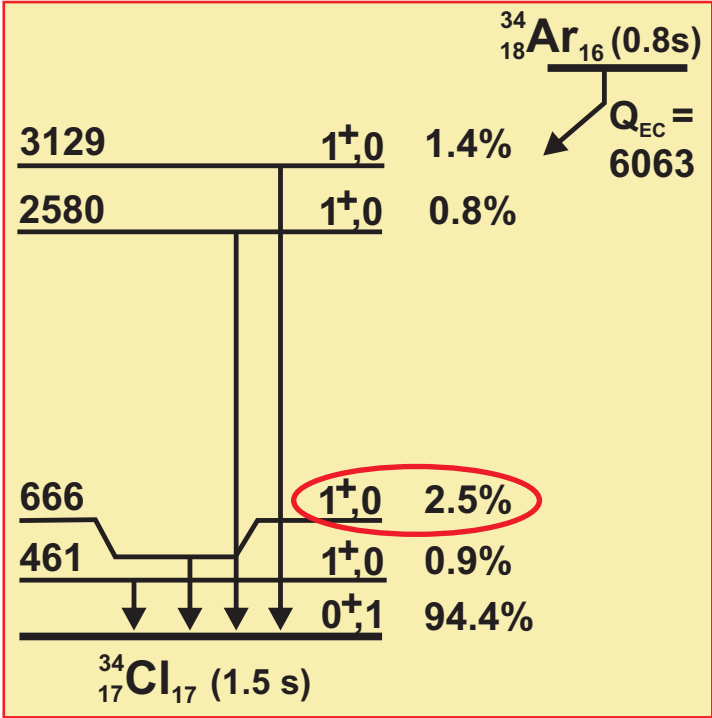


Total-to-peak ratio  
(for summing  
corrections)

511-keV  
escape ratios



# BETA-DECAY BRANCHING OF $^{34}\text{Ar}$

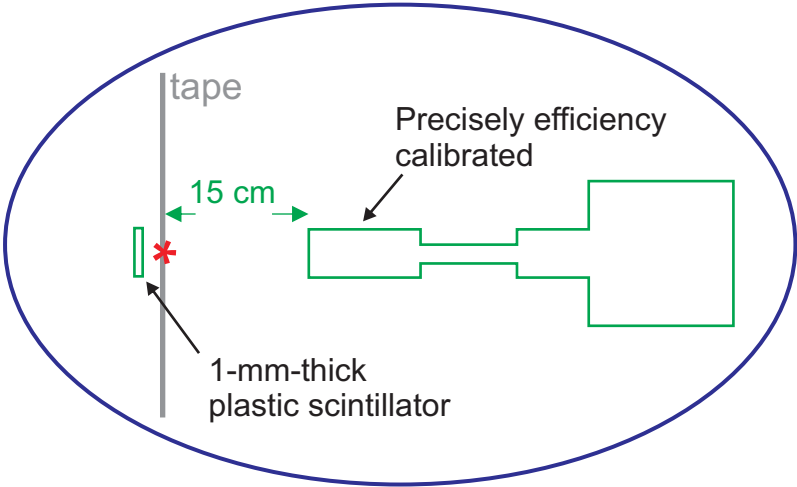
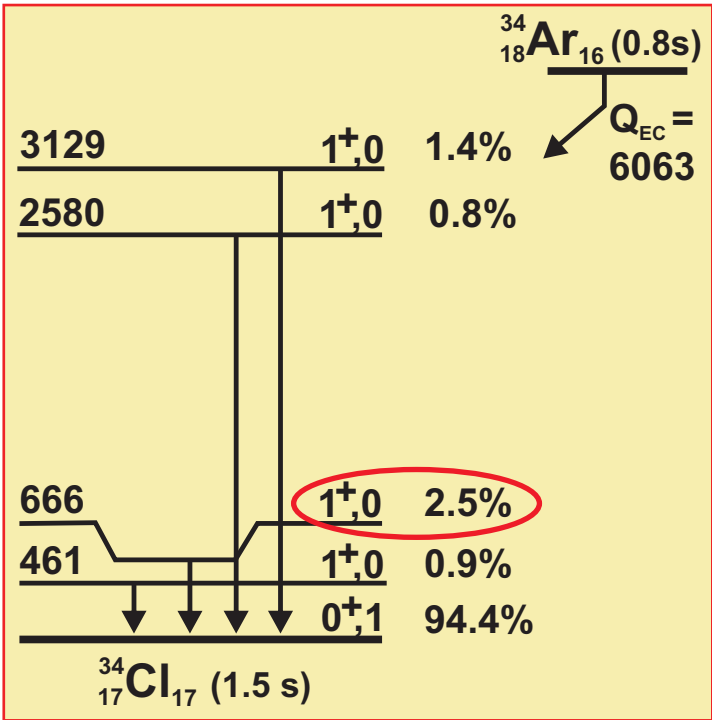


$$\frac{N_1}{N} = \frac{N_0 R_1}{N_0 \text{ tot}}$$

$$R_1 = \frac{N_1}{N} \frac{\text{tot}}{1} k$$

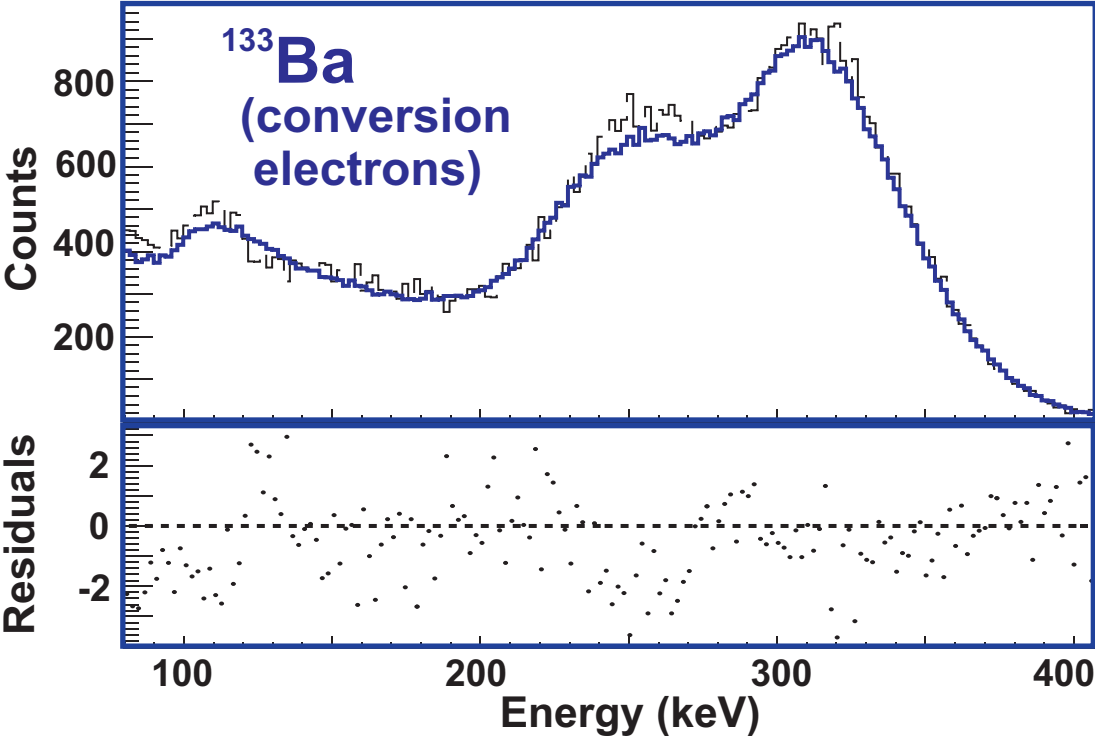


# BETA-DECAY BRANCHING OF $^{34}\text{Ar}$

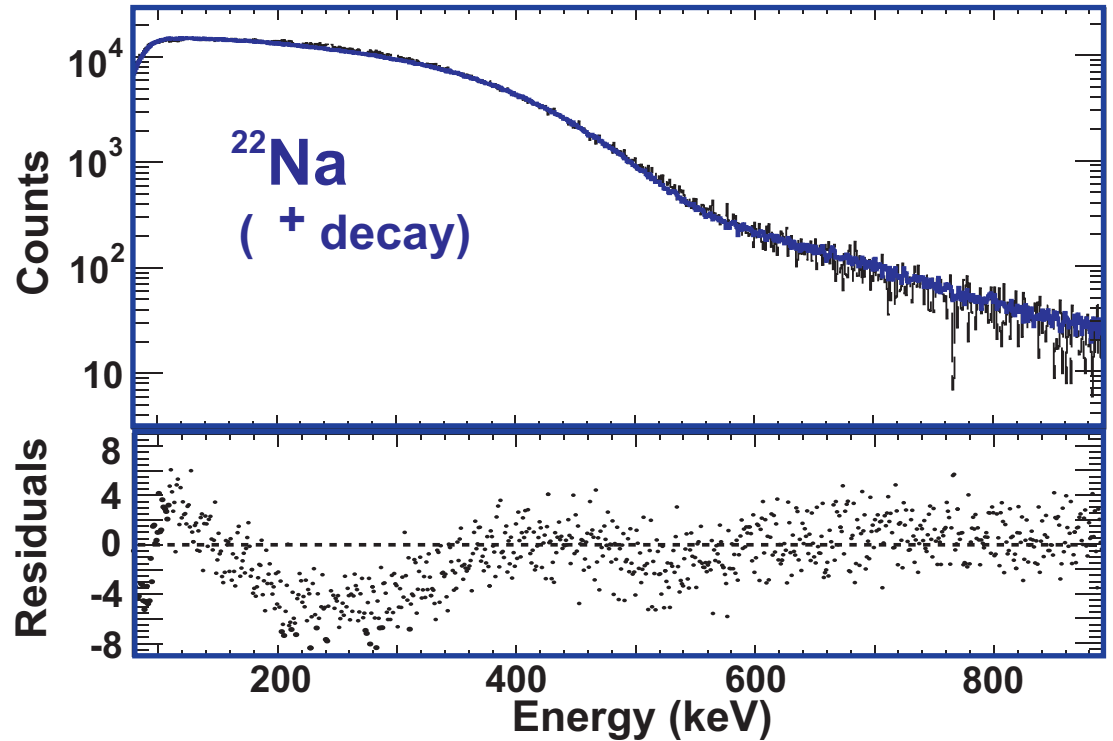
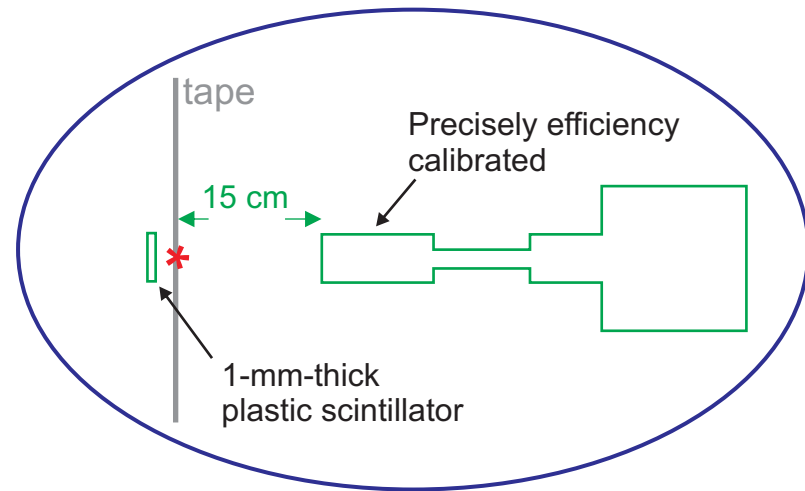
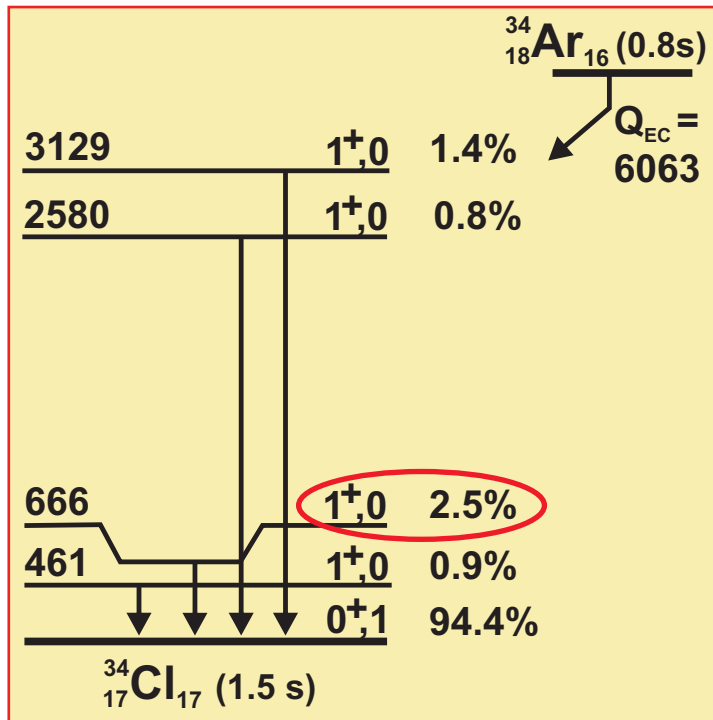


$$\frac{N_1}{N} = \frac{N_0 R_1}{N_0 \text{ tot}}$$

$$R_1 = \frac{N_1}{N} \frac{\text{tot}}{1} k$$



# BETA-DECAY BRANCHING OF $^{34}\text{Ar}$



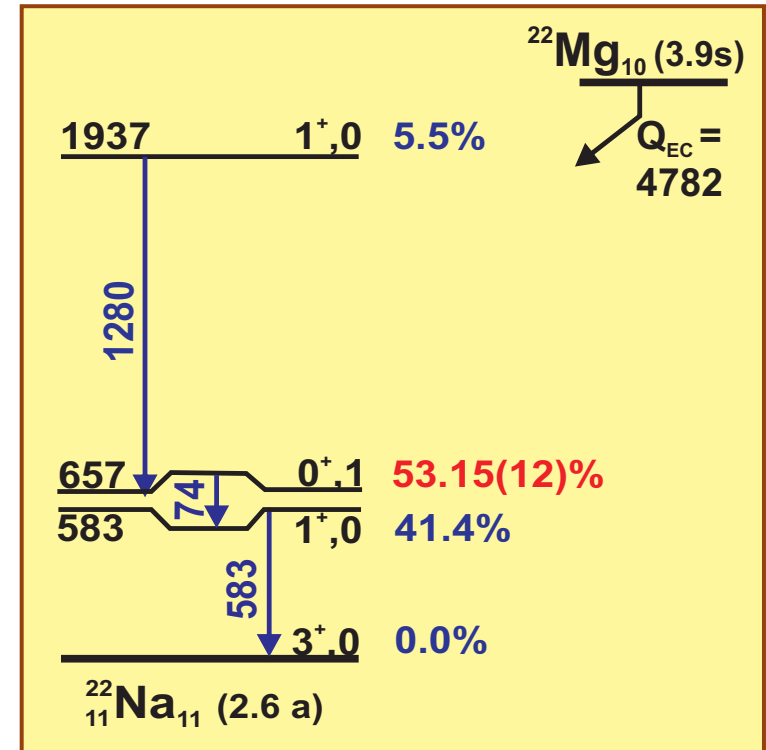
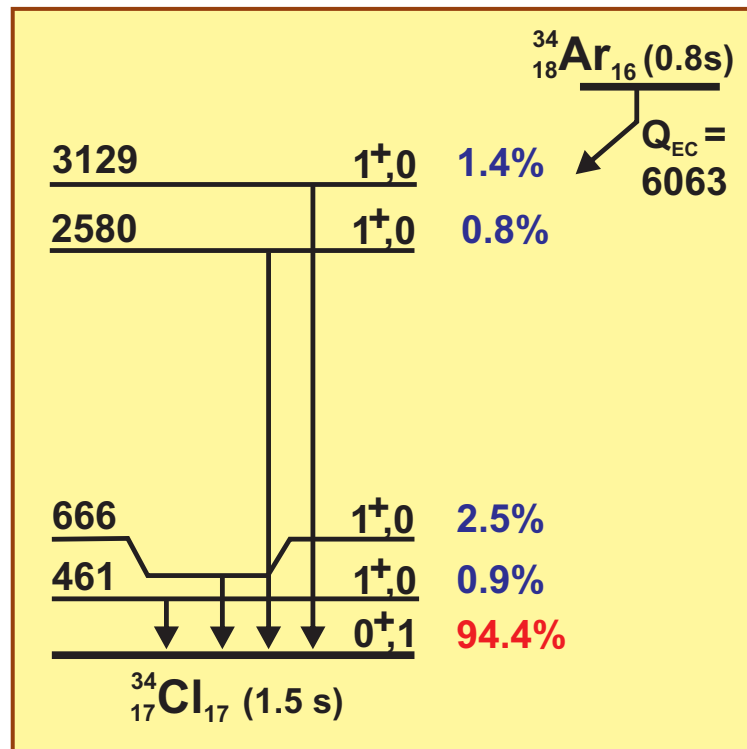
$$\frac{N_1}{N} = \frac{N_0 R_1}{N_0 \text{ tot}}$$

$$R_1 = \frac{N_1}{N} \frac{\text{tot}}{1} k$$

# BRANCHING-RATIO RESULTS

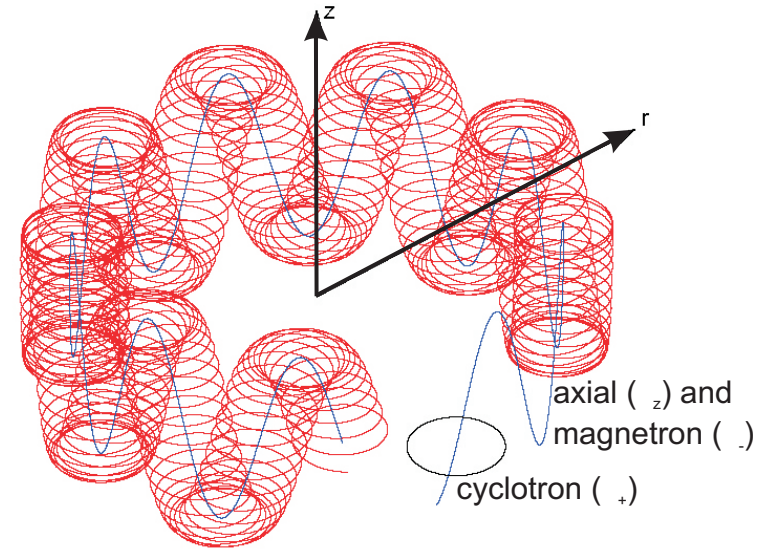
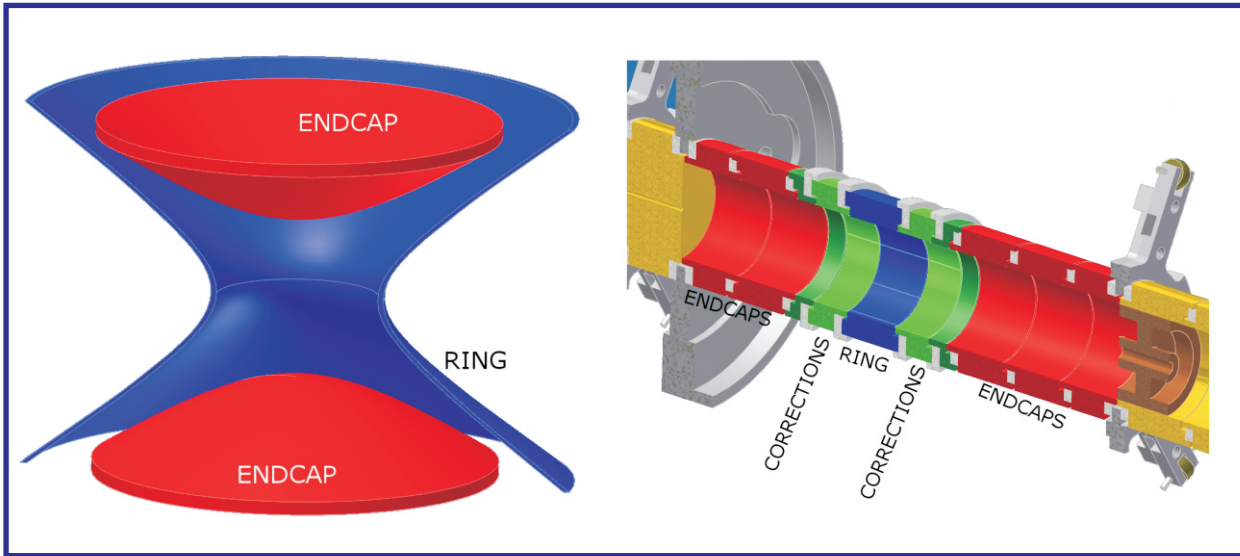
Where no ground-state decay occurs, a  $\gamma$ -ray spectrum and relative efficiencies are enough to obtain branching ratios to  $\pm 0.2\%$ .

Hardy *et al.*, PRL 91, 082501 (2003).

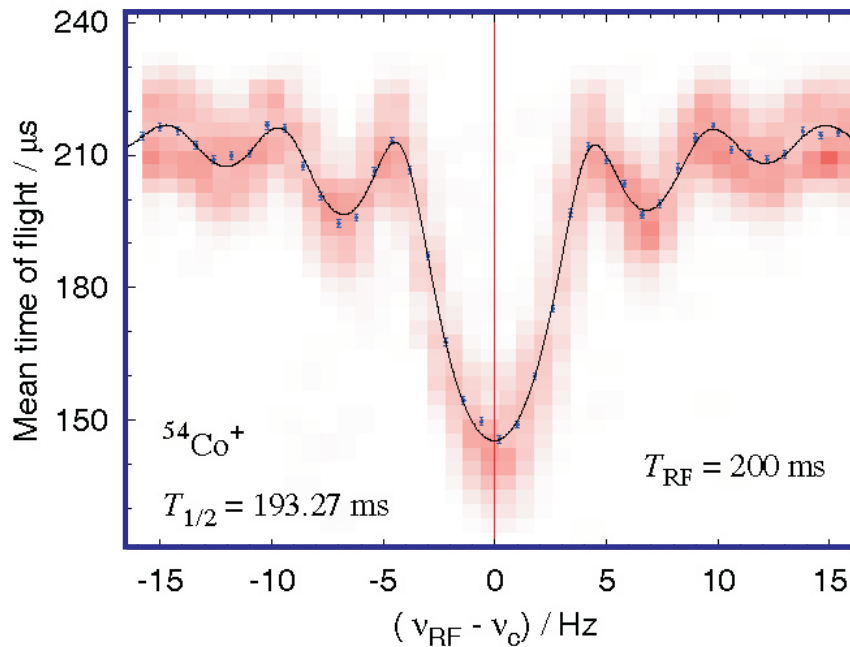


Where superallowed branch feeds the ground state, we measure the other branching ratios to  $\pm 0.2\%$  and subtract them from 100%. In favorable cases (like  $^{34}\text{Ar}$ ) the result can be good to  $\pm 0.01\%$ .

# PENNING TRAP $Q_{EC}$ -VALUE MEASUREMENTS



Time of flight of extracted ions as function of quadrupole rf excitation



$$+ \quad - \quad = \quad c = \frac{1}{2} \frac{q}{m} B$$

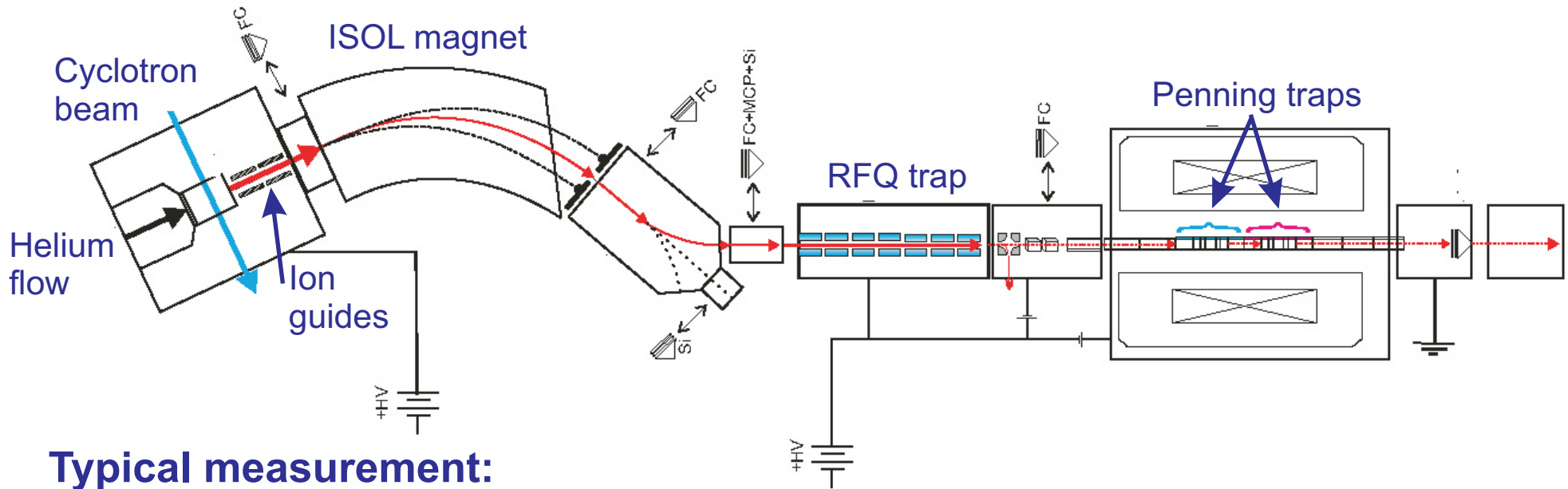
$$Q_{EC} = M_p - M_d$$

$$= \left( \frac{c,d}{c,p} - 1 \right) (M_d - m_e)$$

**Precision,  $Q/Q = 0.001\%$**

# PENNING TRAP $Q_{EC}$ -VALUE MEASUREMENTS

## IGISOL System



Typical measurement:

| <u>Target</u> | <u>E(proton)</u> | <u>Reaction</u>   | <u>Products</u>  |
|---------------|------------------|---|--|
| KCl           | 15 MeV           | $^{35}\text{Cl}(p, 2n)$<br>$^{35}\text{Cl}(p, pn)$<br>$^{35}\text{Cl}(p, 2p)$ | $^{34}\text{Ar}$<br>$^{34}\text{Cl} + ^{34}\text{Cl}^m$<br>$^{34}\text{S}$ |

Eronen *et al.*, Phys. Rev. 83, 055501 (2011)

Or for the full Penning trap Q-value story:  
Tommi Eronen, Jyväskylä Research Report No. 12/2008 (Thesis)

$^{34}\text{Ar}$   $Q_{EC}$  value:  
6061.83(8) keV